Introduction to Modern Physics

Two pillars of modern physics: quantum mechanics (QM) and relativity

both → generalization of classical
physics,
include classical laws
as special cases

o) relativity → extends the range of application
of physical laws to high velocities

universal constant: speed of light $\boxed{c}$

o) QM → extends that range to the
region of small dimensions

universal constant: Planck's constant $\boxed{h}$, $\hbar = h/2\pi$

We start with the milestones that led to modern quantum mech.

Several phenomena in contradiction with classical laws
↳ solving conflicts needed introduction of quantum ideas
Ex: discreteness of energy

concept introduced
by Planck when
studying thermal radiation

Thermal radiation

•) radiation falls on object ⟶ part reflected (more on light-colored bodies)
⟶ part absorbed (more on dark bodies)

•) radiation absorbed ⇒ increases kinetic energy of atoms ⇒ oscillate ⇒
⇒ oscillating/accelerated charges emit

⇓ temperature rises { average translational kinetic energy of atoms determine temperature of body

⇓ temperature reduces { reduces kinetic energy of oscillations

good absorber → good emitter

•) When rate of absorption = rate of emission ⇒ constant temperature
body in thermal equilibrium

↳ electromagnetic radiation emitted called thermal radiation

At ordinary temperatures (< 600°C): thermal radiation NOT visible
large λ, short ν
object seen by reflected light

600°C – 700°C : enough energy in visible spectrum, object glows

→ higher T → body emits more, ν becomes higher

Detailed form of the spectrum of the thermal radiation emitted deps. on composition but for a
black body it is universal and depends only on the temperature
↳ ideal body that absorbs all thermal radiation incident upon it

→ all blackbodies at the same temperature emit thermal radiation with the same spectrum.

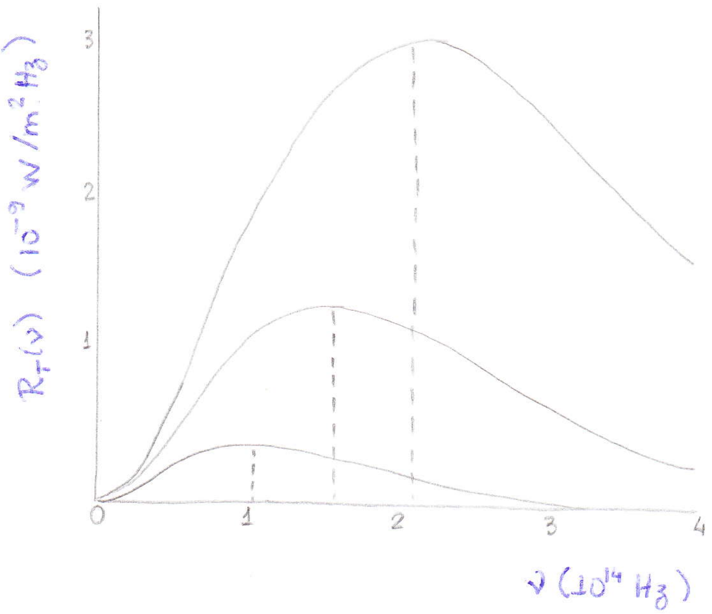
Specific form of a <u>blackbody</u> spectrum <u>CANNOT</u> be understood on the basis of <u>classical</u> arguments

Spectral distribution of blackbody radiation

$R_T(\nu)$ = spectral radiancy $\left\{ \begin{array}{l} \text{energy emitted per unit time, per unit area} \\ \text{per unit frequency} \rightarrow \frac{J}{s\,m^2\,Hz} = \frac{W}{m^2\,Hz} \end{array} \right.$

$R_T(\nu)\,d\nu = \underbrace{\text{energy emitted per unit time}}_{(\text{power}) \rightarrow ⓦ}$ in radiation of frequency in the interval ν to $\nu + d\nu$ per unit area of the surface at temperature T

<u>experiments</u>:



(1) power decreases as $\nu \rightarrow 0$, $\nu \rightarrow \infty$

(2) <u>frequency</u> at which radiated power is most intense increases with <u>temperature</u>

$\boxed{\nu_{max} \propto T}$ Wien's ⊛ displacement law

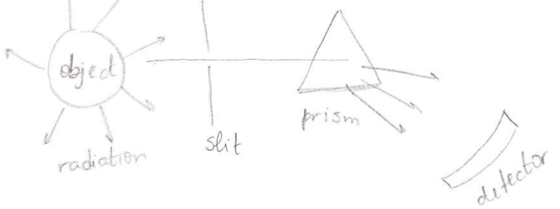
(3) total <u>power</u> radiated in all frequencies increases with <u>temperature</u>

$$R_T = \int_0^\infty R_T(\nu)\,d\nu \quad \left(\begin{array}{l}\text{area under} \\ \text{the curve}\end{array}\right)$$

↳ radiancy = flux of $\underbrace{\text{energy}}_{\boxed{\text{intensity}}}$ = energy per unit time per unit area $\left(\frac{J}{m^2 s} = \frac{W}{m^2}\right)$

$\boxed{R_T = \sigma T^4}$ Stefan's law

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$ (Stefan - Boltzmann const)



⊛ <u>Obs</u>: $\lambda \nu = c \Rightarrow \boxed{\lambda_{max} T = \overset{\text{Wien's constant}}{2.898 \times 10^{-3}\ m\,K}}$

Question

Assume that stellar surfaces behave like blackbodies.
For the sun $\lambda_{max} = 5100$ Å, whereas for the North Star $\lambda_{max} = 3500$ Å
Find the surface temperature of these stars
(1 Å = 10^{-10} m)

$$\lambda_{max} T = 2.898\times10^{-3} \text{ m.K}$$

$$T_{sun} = \frac{2.898\times10^{-3}}{5100\times10^{-10}} = \boxed{5700\text{K}} // \qquad T_{\substack{North\\star}} = \frac{2.898\times10^{-3}}{3500\times10^{-10}} = \boxed{8300\text{K}} //$$

Question

Measurement of λ_{max} from a certain star indicates $T = 3000$K. If the star radiates 100 times the power radiated by the sun, how big is the stare? Sun surface temperature = 5800K

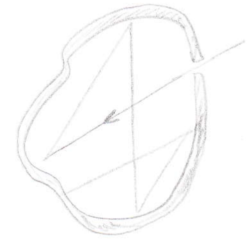
$$\left.\begin{array}{l} R_{star} = \dfrac{P_{star}}{area_{star}} = \dfrac{100\, P_{\odot}}{4\pi r^2_{star}} = \sigma T^4_{star} \Rightarrow 4\pi r^2_{star} = \dfrac{100\, P_{\odot}}{\sigma T^4_{star}} \\[2em] R_{\odot} = \dfrac{P_{\odot}}{area_{\odot}} = \dfrac{P_{\odot}}{4\pi r^2_{\odot}} = \sigma T^4_{\odot} \Rightarrow P_{\odot} = \sigma T^4_{\odot}\,(4\pi r^2_{\odot}) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \cancel{4\pi} r^2_{star} = \frac{100\, \cancel{\sigma} T^4_{\odot}\,(\cancel{4\pi} r^2_{\odot})}{\cancel{\sigma} T^4_{star}} \Rightarrow r_{star} = 10\, r_{\odot} \left(\frac{T_{\odot}}{T_{star}}\right)^2$$

$$r_{star} = 10\left(\frac{5800}{3000}\right)^2 r_{\odot} \Rightarrow \boxed{r_{star} = 37.4\, r_{\odot}} //$$

∘) CLASSICAL theory of cavity radiation

Example of blackbody : a cavity in a body connected by a small hole to the outside



(lab blackbody)

energy density $\rho_T(\nu)$ = energy contained in a unit volume of the cavity at temperature T

$\rho_T(\nu) \propto R_T(\nu)$

→ [ⓤ in Tipler]

Energy per unit volume in the frequency interval ν to $\nu + d\nu$

Rayleigh and Jeans

$$\rho_T(\nu)\,d\nu = \frac{\bar{\mathcal{E}}\,N(\nu)\,d\nu}{V}$$

$N(\nu)\,d\nu$ → number of standing waves in the frequency interval ν to $\nu + d\nu$

$\bar{\mathcal{E}}$ → average energy of each standing wave in cavity

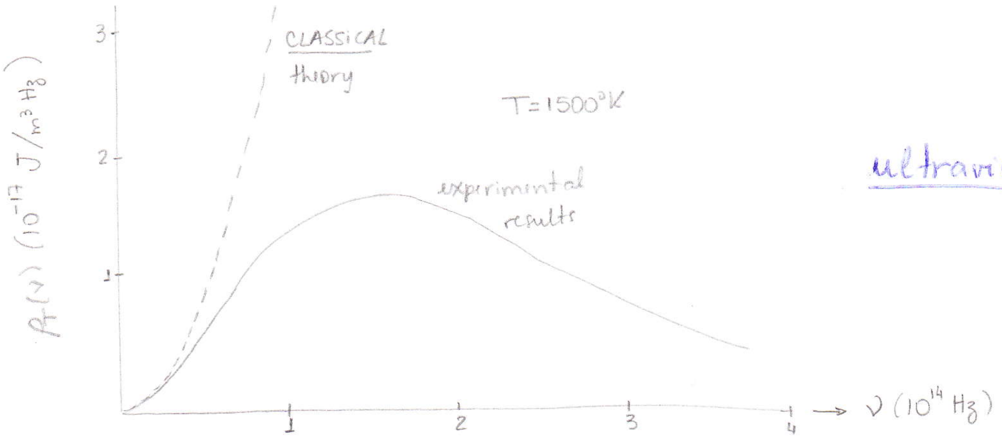
V → volume

$$N(\nu)\,d\nu = \frac{8\pi V}{c^3}\,\nu^2\,d\nu$$ ← derivation from geometrical arguments (see Eisberg's book)

Classical equipartition law $\bar{\mathcal{E}} = KT$

$$\boxed{\rho_T(\nu)\,d\nu = \frac{8\pi\,\nu^2 KT}{c^3}\,d\nu}$$

Rayleigh-Jeans formula for black body radiation



ultraviolet catastrophe

Classical equipartition law

Boltzmann distibution (appendix C - Eisberg's book)

$P(\mathcal{E}) = A e^{-\mathcal{E}/KT}$

$\int_0^\infty P(\mathcal{E})\, d\mathcal{E} = 1 \Rightarrow \boxed{A = \frac{1}{KT}}$

$$P(\mathcal{E}) = \frac{e^{-\mathcal{E}/KT}}{KT}$$

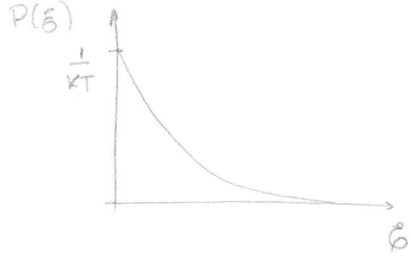
$e^{-\mathcal{E}/KT}$ → fraction of oscillators with energy $\mathcal{E}$

KT → normalization → guarantees that $\boxed{\int_0^\infty P(\mathcal{E})\, d\mathcal{E} = 1}$

(standing wave, ..)

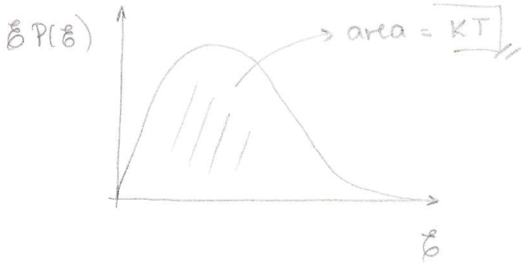
$P(\mathcal{E})\, d\mathcal{E}$: probability of finding an entity of the system with energy in $[\mathcal{E}, \mathcal{E}+d\mathcal{E}]$

↳ system with large number of entities in thermal equilibrium at T



→ large prob. for $\mathcal{E} = 0$

average energy. $\bar{\mathcal{E}} = \int_0^\infty \mathcal{E}\, P(\mathcal{E})\, d\mathcal{E}$



$$\int_0^\infty \frac{\mathcal{E} e^{-\mathcal{E}/KT}}{KT}\, d\mathcal{E} =$$

by parts

$u = \mathcal{E}$ $\quad dv = e^{-\mathcal{E}/KT}\, d\mathcal{E}$

$du = d\mathcal{E}$ $\quad v = e^{-\mathcal{E}/KT}(-KT)$

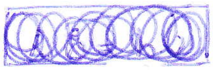
$$= \frac{1}{KT}\left[\cancel{\mathcal{E} e^{-\mathcal{E}/KT}(-KT)\Big|_0^\infty} - \int_0^\infty e^{-\mathcal{E}/KT}(-KT)\, d\mathcal{E} \right] =$$

$$= \int_0^\infty e^{-\mathcal{E}/KT}\, d\mathcal{E} = e^{-\mathcal{E}/KT}(-KT)\Big|_0^\infty = \boxed{KT}$$

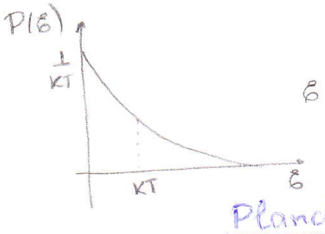
↳ <u>Integration</u> over <u>ALL</u> possible energies

↳ here $\mathcal{E}$ is a <u>CONTINUOUS</u> variable

•) Planck's theory of cavity radiation $\rho_T(\nu)\,d\nu = \frac{8\pi\nu^2\,\bar{\mathcal{E}}}{c^3}\,d\nu$



$$\left.\begin{array}{l}\bar{\mathcal{E}} \xrightarrow[\nu\to 0]{} KT \\ \bar{\mathcal{E}} \xrightarrow[\nu\to\infty]{} 0\end{array}\right\}$$ to avoid discrepancy with experimental results, energy needs to be a function of ν



$\mathcal{E} \gg KT$ little contribution to $\bar{\mathcal{E}}$

$\boxed{\mathcal{E} = nh\nu}$ DISCRETE

Planck's constant $\boxed{h = 6.63\times10^{-34}\ \text{J.s}}$

determined by best fit of theory with experiment

$\boxed{\mathcal{E} = nh\nu}$ $n = 0, 1, 2, 3 \ldots$ → energy is discrete / QUANTIZED

$$\bar{\mathcal{E}} = \sum \mathcal{E} \underset{\substack{\uparrow\\ \text{normalization}}}{A}\, e^{-\mathcal{E}/KT} \qquad \sum A e^{-\mathcal{E}/KT} = 1$$

$$\hookrightarrow \boxed{A = \frac{1}{\sum e^{-nh\nu/KT}}}$$

$$\bar{\mathcal{E}} = \frac{\sum_{n=0}^{\infty} \frac{nh\nu}{KT}\, e^{-nh\nu/KT}}{\sum_{n=0}^{\infty} [illegible]\ e^{-nh\nu/KT}}\ (KT)$$

One way to solve

$$\sum_{n=0}^{\infty} e^{-nx} = 1 + e^{-x} + e^{-2x} + \ldots = \frac{1}{(1-e^{-x})}$$

$$\sum_{n=0}^{\infty} n x e^{-nx} = xe^{-x} + 2xe^{-2x} + 3xe^{-3x} + \ldots$$

$$= xe^{-x} + xe^{-2x} + xe^{-3x} + xe^{-4x} + \ldots$$
$$+ xe^{-2x} + xe^{-3x} + xe^{-4x} + \ldots$$
$$+ xe^{-3x} + xe^{-4x} + \ldots$$
$$\ldots$$

$$= xe^{-x}(1 + e^{-x} + e^{-2x} + \ldots) + xe^{-2x}(1 + e^{-x} + \ldots) + xe^{-3x}(1 + \ldots)$$

$$= \frac{1}{(1-e^{-x})}\, xe^{-x}(1 + e^{-x} + e^{-2x} + \ldots) = \frac{xe^{-x}}{(1-e^{-x})^2}$$

$$\boxed{\frac{\sum nxe^{-nx}}{\sum e^{-nx}} = \frac{xe^{-x}}{(1-e^{-x})}}$$

Another way to solve (Eisberg)

Note:

$$\frac{\sum_{n=0}^{\infty} nxe^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = -x\frac{d}{dx}\ln\sum_{n=0}^{\infty} e^{-nx}$$

since

$$= -x\frac{\frac{d}{dx}\sum_{n=0}^{\infty} e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = -\frac{\sum_{n=0}^{\infty} x\frac{d}{dx}(e^{-nx})}{\sum_{n=0}^{\infty} e^{-nx}} = \frac{\sum nxe^{-nx}}{\sum e^{-nx}}$$

$$-x\frac{d}{dx}\ln\sum_{n=0}^{\infty} e^{-nx} = -x\frac{d}{x}\ln\left(\frac{1}{1-e^{-x}}\right)$$

$$= -x\,(1-e^{-x})\frac{(-1)}{(1-e^{-x})^2}\,e^{-x} = \boxed{\frac{xe^{-x}}{(1-e^{-x})}}$$

$$\Rightarrow \bar{\mathcal{E}} = KT\left(\frac{h\nu}{KT}\right)\frac{e^{-h\nu/KT}}{(1-e^{-h\nu/KT})}$$

$x = h\nu/KT$

$$\boxed{\bar{\mathcal{E}} = \frac{h\nu}{e^{h\nu/KT} - 1}}$$ //

$$\bar{\mathcal{E}}(\nu) = \frac{h\nu}{e^{h\nu/kT}-1} \quad \begin{cases} \nu \to 0 \Rightarrow \bar{\mathcal{E}} \to \dfrac{h\nu}{1+\frac{h\nu}{kT}+\cdots-1} \to \boxed{kT} \\ \nu \to \infty \Rightarrow \bar{\mathcal{E}} \to h\nu\, e^{-h\nu/kT} \to \boxed{0} \end{cases}$$

$$\boxed{\rho_T(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\,\frac{h\nu}{e^{h\nu/kT}-1}\,d\nu}$$

Planck's blackbody spectrum

Planck's blackbody spectrum in terms of λ

$$\boxed{c = \lambda\nu} \Rightarrow \nu = \frac{c}{\lambda} \Rightarrow \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow \boxed{d\nu = -\frac{c}{\lambda^2}\,d\lambda}$$

$$\rho_T(\lambda)\,d\lambda = \ominus\, \rho_T(\nu)\,d\nu$$

$\left\{ \begin{array}{l} d\lambda,\ d\nu \text{ have opposite sign} \\ \text{increase in } \nu \to \text{decrease in } \lambda \end{array} \right.$

$$\rho_T(\lambda)\,d\lambda = \frac{c}{\lambda^2}\,\rho_T(\nu)\,d\lambda = \frac{\cancel{c}}{\lambda^2}\,\frac{8\pi}{\cancel{c^3}}\,\frac{c^2}{\lambda^2}\,\frac{hc/\lambda}{e^{hc/\lambda kT}-1}\,d\lambda$$

$$\boxed{\rho_T(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5}\,\frac{d\lambda}{e^{hc/\lambda kT}-1}}$$

<u>HW</u>

Tipler: 3-12, 13, 14, 15, 16, 17, 20

<u>Question</u>: Show that the total energy density in a blackbody cavity is proportional to T^4 in accordance with Stefan-Boltzmann law

$$U = \int_0^\infty \rho(\nu)\,d\nu = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3}{e^{h\nu/kT}-1}\,d\nu \qquad x = \frac{h\nu}{kT} \qquad dx = \frac{h}{kT}\,d\nu$$

$$U = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^3 \int_0^\infty \frac{x^3}{e^x-1}\left(\frac{kT}{h}\right)dx$$

$$U = \frac{8\pi}{c^3h^3}(kT)^4 \underbrace{\int_0^\infty \frac{x^3}{e^x-1}\,dx}_{(\text{dimensionless}) = \pi^4/15} \Rightarrow \boxed{U \propto T^4}$$

use $U = \dfrac{8\pi^5 k^4 T^4}{15 h^3 c^3}$ and $R = \dfrac{c}{4}U$ to find σ in terms of h, c, k

-) Blackbody radiation → discrete / quantized

  ↳ energy is DISCRETE (Planck 1900)

Big Bang
$T = 2.725 \pm 0.001\ K$

-) Electric Charge is DISCRETE

Greek
→) Democritus (450 B.C.) – matter is composed of atoms (tiny particles)

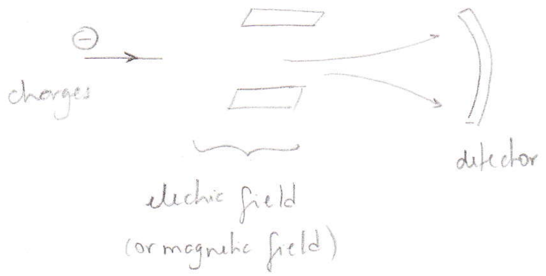
→) Avogadro (1811) – all gases at T: same # of molecules per unit volume (N_A)

↳ development of Kinetic theory (1900) – acceptance of molecular theory of matter

matter is quantized

→) J. J. Thomson's experiment (1897)

↳ measures q/m



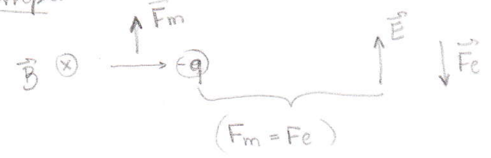
-) Newton's 2nd law

$$\underbrace{q v B}_{\text{magnetic force}} = m \underbrace{\frac{v^2}{R}}_{\text{centripetal acceleration}} \Rightarrow \boxed{\frac{q}{m} = \frac{v}{RB}}$$

-) to determine v
adjust ⊥ B and E fields
↳ particle undeflected

$$qvB = qE \Rightarrow \boxed{v = \frac{E}{B}}$$

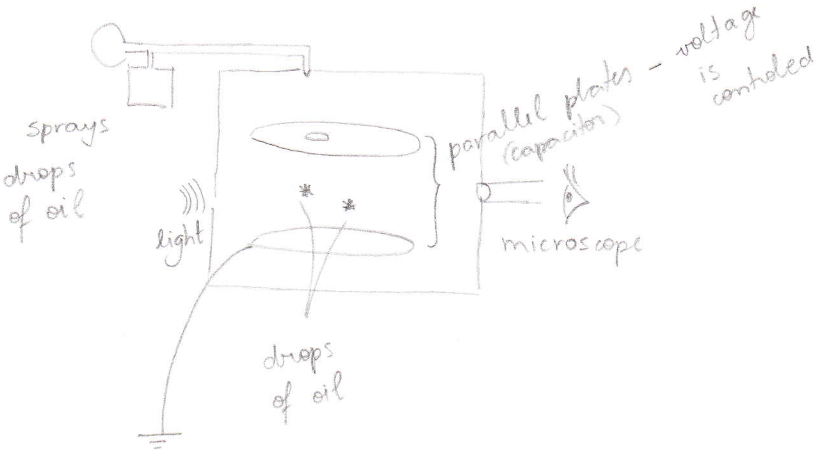
Example



→) Millikan's experiment (1909)

↳ measures e

Millikan's experiment



(oil droplets don't evaporate easily)

we verify that the charge of each droplet is an integer number times the electron charge

(electric charge is quantized)

Assumption: oil-drops = spherical droplets of const m

↳ gravitational force $\boxed{F_g = mg} = \frac{4}{3}\pi \underset{\text{radius}}{r^3} \overset{\text{density}}{\rho} g$

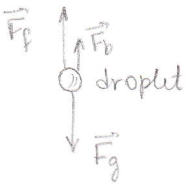
1st step (find r)

$\vec{E} = 0$, a const velocity of fall is reached when

$F_g = \boxed{F_f} \rightarrow$ friction force given by Stoke's law $\qquad \boxed{F_f = 6\pi r \overset{\text{viscosity of air}}{\eta} \overset{\text{speed of droplet}}{v}}$

correction!

BUT, because of the buoyant force $F_b = \underset{\text{mass of the fluid displaced}}{\boxed{m_F}} g = \frac{4}{3}\pi r^3 \rho_{air} g$



$$\boxed{F_g - F_b = F_f} \implies \frac{4}{3}\pi r^3 g (\rho - \rho_{air}) = 6\pi r \eta v_1$$

$$\boxed{r^2 = \frac{9\eta v_1}{2g(\rho - \rho_{air})}}$$

2nd step
(find q)

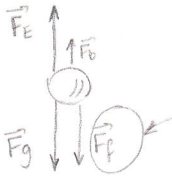
$\vec{E} \neq 0$, drop rises

$$E = \frac{V}{d}$$

we have control of the voltage

d ← distance between plates

$$\boxed{F_E = qE}$$



notice that friction opposes the motion

Drop reaches const velocity when

$$F_E + F_b = F_g + F_f$$

$$qE = \underbrace{\underbrace{(F_g - F_b)}_{6\pi r \eta v_1}}_{\text{from 1st step}} + \underbrace{F_f}_{6\pi r \eta v_2}$$

$$\boxed{q = \frac{6\pi r \eta (v_1 + v_2)}{E}}$$

HW (Tipler: 3.1, 3.2, 3.3, 3.4, 3.8, 3.11)

Photoelectric effect :

ejection of electrons from a surface by the action of light

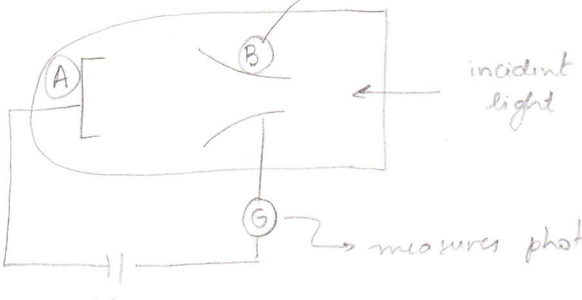
(solar cells)

Hertz (1886/1887) : studies of electromagnetic waves
Maxwell's electromagnetic theory of light propogation

↳ noticed electric discharge between electrodes for UV ⓥ

↑

Lenard continued studies



→ attracts the electrons ejected by Ⓐ

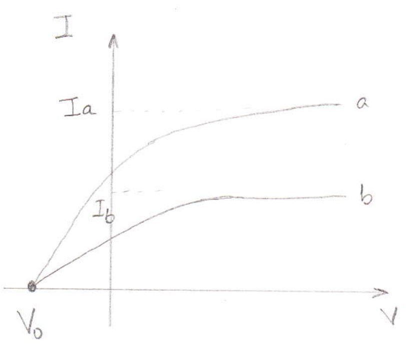
incident light

↳ measures photoelectric current

V potential difference
↳ may be varied

Observations

→) V large : current reaches a limiting value / saturates
all photoelectrons ejected from A are collected by B



→) sign of V is reversed : current does not drop immediately ⇒
⇒ e^- from A are emitted with Kinetic energy

↳ only for large value of reversed of the potential difference $\boxed{V_0}$ will the current drop to zero

$$\boxed{K_{max} = e V_0}$$

↳ Kinetic energy of the fastest e^-

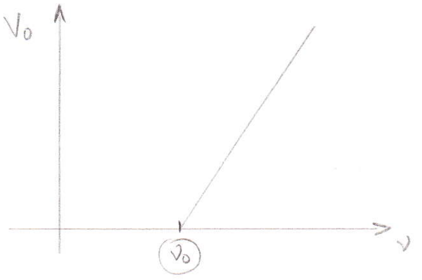
! <u>1st surprise</u> : V_0 (and therefore K_{max}) is <u>independent</u> of the <u>intensity</u> of the light

(compare curves ⓐ and ⓑ)

{ from classical/wave theory we would expect K_{max} to increase with I, since

I increases $\Rightarrow |\vec{E}|$ increases $\Rightarrow$ force eE applied on e^- should increase

! <u>2nd surprise</u> :



value is characteristic of the surface

there is a <u>cutoff</u> frequency (ν_0) below which <u>no</u> photoelectric effect occurs, no matter how intense the radiation

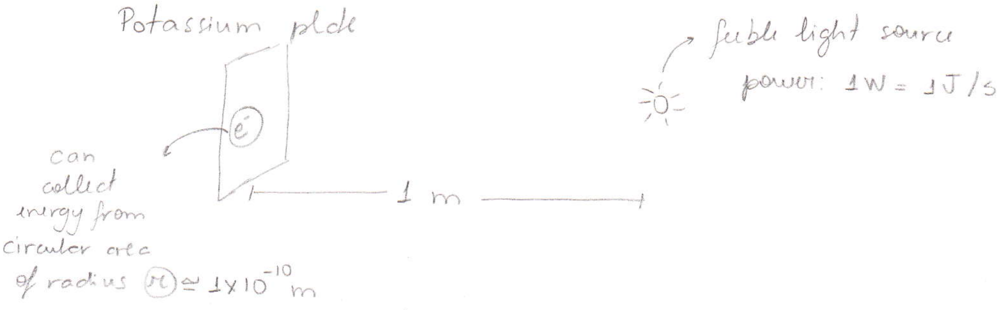
{ but from classical theory we would expect that the effect should occur for any ν if light was intense enough

! <u>3rd surprise</u> : there is <u>no</u> detectable <u>time lag</u>

{ from classical theory, if the light is feeble there should be a time lag between the moment light starts to impinge on the surface and the ejection of the e^-

e^- would need a time to accumulate enough energy to escape

Example.



Energy to remove the $e^- = 2.1$ eV $= 3.4 \times 10^{-19}$ J
(work function)

(1 eV = 1.60×10^{-19} J is the energy gained by an e^- when it accelerates through a potential difference of 1 V)

Obs

$V = E\,d$ (potential difference)

$U = q\,V$ (potential energy)

According to classical theory
how long would it take
for the target to absorb
enough energy from the source?

target area: $\pi r^2 = \pi \times 10^{-2}\,m^2$

area of sphere centered at source: $4\pi\,(1m)^2$

$$\left.\begin{array}{lll} \text{power} = 1\,W & \text{for} & 4\pi\,m^2 \\ x & \text{for} & \pi \times 10^{-2}\,m^2 \end{array}\right\} \quad x = 1\,\frac{J}{s}\;\frac{\cancel{\pi} \times 10^{-2}\,\cancel{m^2}}{4\cancel{\pi}\,\cancel{m^2}} = \boxed{2.5 \times 10^{-21}\,J/s}$$

power absorbed by target

$$\left.\begin{array}{lll} 1\,s & \longrightarrow & 2.5 \times 10^{-21}\,J \\ x & \longrightarrow & 3.4 \times 10^{-19}\,J \end{array}\right\} \quad x = \frac{3.4 \times 10^{-19}\,J\;(1s)}{2.5 \times 10^{-21}\,J} = 1.4 \times 10^{2}\,s \simeq \underline{\underline{2\,\text{min}}}$$

Einstein' quantum theory of the photoelectric effect

Planck: energy quantization for blackbody

Einstein: extended the idea → energy is quantized into lumps → photons

$\boxed{E = h\nu}$ → energy of photon

He assumed { In the photoelectric process, one photon is completely absorbed by one electron

e^- emitted with kinetic energy: $\boxed{K = h\nu - w}$

↳ work to remove e^- from metal

↳ energy of the absorbed incident photon

for the loosest binding

$\boxed{K_{max} = h\nu - w_0}$

↳ characteristic of metal

minimum energy for e^- to escape

Conclusions:

1) Kmax has no dependence on I

(increasing I ⇒ only increases # of photons and current;

↳ it DOES NOT change energy hν of photon

2) cutoff is justified

$K_{max} = 0 \Rightarrow \boxed{h\nu_0 = w_0}$

$\nu < \nu_0 \Rightarrow$ no matter how many photons, how intense the source ⇒ NO ejection

3) energy of photon is concentrated and NOT spread

photon hits surface / is immediately absorbed

e^- is " emitted

$$\left.\begin{array}{l} K_{max} = eV_0 \\ \boxed{K_{max} = h\nu - w_0} \end{array}\right\} \quad \boxed{V_0 = \frac{h}{e}\nu - \frac{w_0}{e}}$$

Einstein

$\Downarrow$

stopping potential

linear relationship between V_0 and ν

(agrees with experiments)

$$\left.\begin{array}{l} \text{slope of experimental curve} \Rightarrow \frac{h}{e} = 4.0\times10^{-15}\ V s \\ \\ \boxed{h = 6.6262\times10^{-34}\ J.s} \end{array}\right\} \quad 1V = \frac{1J}{C}$$

which agrees with h derived from Planck's radiation formula (!)

(Numerical agreement of h for 2 different phenomena)

<u>Exercise</u>: λ for visible light 380nm — 750nm

a) what is the range of photon energies (in eV) in visible light?

$1\,nm = 10^{-9}\,m$ $\qquad c = \lambda\nu$

$1\,eV = 1.60\times10^{-19}\,J$

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6262\times10^{-34}\,(J s)\ 3\times10^{8}\,(m/s)}{\lambda}$$

$$E = \frac{6.6262\times10^{-34}}{1.60\times10^{-19}}(eV.s)\ \frac{3\times10^{8}}{10^{-9}}\ (nm/s)\ \frac{1}{\lambda}$$

$$\boxed{E = \frac{1242\ (eV.nm)}{\lambda}}$$

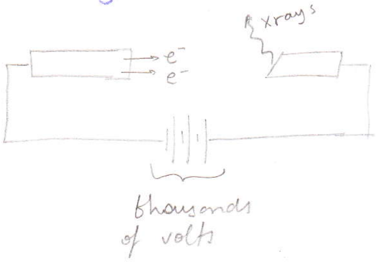
$\lambda = 380nm \Rightarrow E = 3.27\,eV$

$\lambda = 750nm \Rightarrow E = 1.66\,eV$

<u>HW</u>: 3.26, 3.28, 3.30

X - Rays

W. K. Roentgen (1895)



X rays
- •) no deflection in $\vec{B}$
- •) was not able to see diffraction, interference
- •) most materials – transparent to it

↓ decreases with material density

↓ found medical application (activate photographic film)

hard to see diffraction (bending of waves caused by obstacle) because $\lambda = 1Å$

↳ need slit $\sim 1Å$

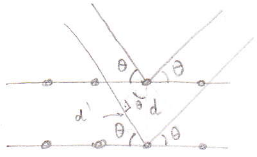
↳ crystal (array of atoms separated $\sim 1Å$) ⇒ diffraction

↓

confirmed that X rays are electromagnetic waves

Bragg (1912)

↳ use of x rays to determine the structure of crystals



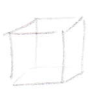
- •) treat atomic plates as reflecting mirrors
- •) assume ∡ of incidence = ∡ of reflection

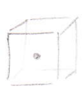
$\sin\theta = \frac{d'}{d}$ path difference: $\Delta = 2d' = 2d\sin\theta$

interference is constructive if $\Delta = n\lambda$ $n = 1, 2, 3 \ldots$

$$\boxed{2d\sin\theta = n\lambda}$$ Bragg's law

λ is known, θ from experiment ⇒ can obtain the interplanar distance d

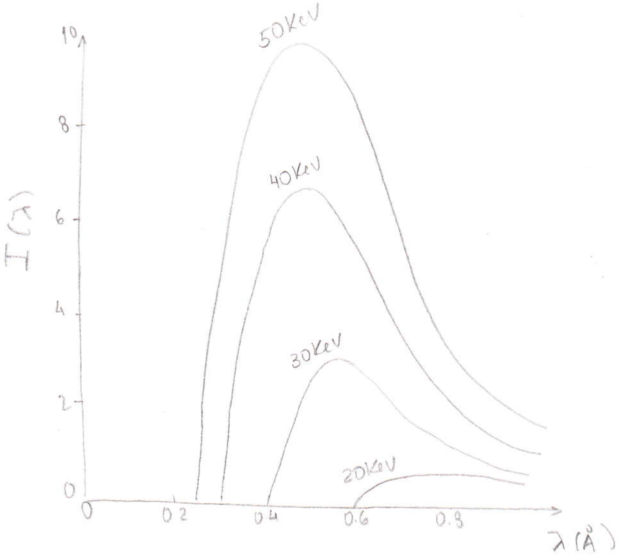

sc


bcc


fcc

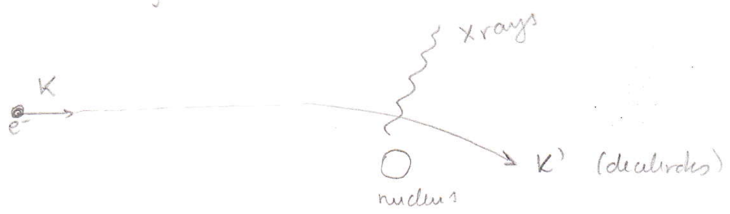
Study of the X-ray spectrum

↳ depends on the accelerating voltage and material



(1) continuous X radiation

$\underbrace{\text{brems}}_{\text{decelerating}}\ \underbrace{\text{strahlung}}_{\text{radiation}}$



(2) sharp lines (NOT shown)

(3) cutoff λmin

↳ indep. of material

dep on energy of e⁻ / the potential

$h\nu = K - K'$ { an inverse photoelectric effect

photoelectric ↓ photon absorbed / E, p go to e⁻

$\frac{hc}{\lambda} = K - K'$ bremsstrahlung: photon created / E, p from e⁻ colliding with nucleus

$K' = 0 \Rightarrow \boxed{K = \frac{hc}{\lambda_{min}}}$ (electron loses all its kinetic energy)

$K = eV \Rightarrow eV = \frac{hc}{\lambda_{min}} \Rightarrow \boxed{\lambda_{min} = \frac{hc}{e}\frac{1}{V}}$ ← voltage

$$\frac{1240\,(eV)\,(nm)}{1.60 \times 10^{-19}\,C}$$

$$\frac{1240 \times 1.60 \cancel{\times 10^{-19}}\,J}{1.60 \cancel{\times 10^{-19}}\,C}\ (nm) = 1.24 \times 10^{3}\ \overset{\text{unit}}{V}\cdot nm$$

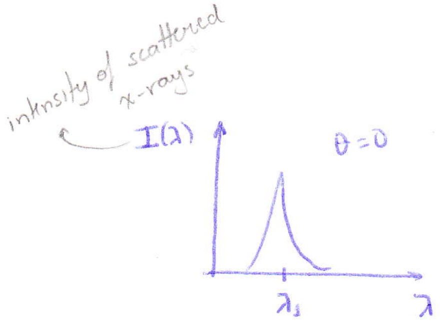
Exercise: Determine h from the fact that λmin = 3.11×10⁻¹¹ m for 400 KeV electrons.

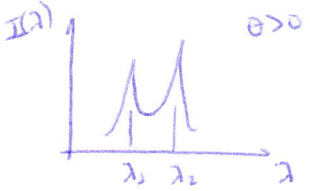
$K = h\nu = \frac{hc}{\lambda} \Rightarrow h = \frac{K\lambda}{c}$

$h = \frac{400 \times 10^{3}\,eV \times 1.6 \times 10^{-19}\,J \times 3.11 \times 10^{-11}\,m}{3 \times 10^{8}\,m/s}$

$= \boxed{6.64 \times 10^{-34}\,J.s}$

Compton Effect





incoming x-ray not a wave, but collection of photons with E = hν. They collide with free e⁻

$\Delta\lambda = \lambda_2 - \lambda_1 \rightarrow$ Compton shift → quantum effect

classically: oscillating $\vec{E} \Rightarrow e^-$ oscillates at the same ν ⇒ can only explain λ_1

$$\boxed{\lambda_2 - \lambda_1 = \underbrace{\frac{h}{mc}}_{0.00243\,nm}(1-\cos\theta)}$$

→ explains λ_2 (collision with free electrons) (θ > 0)

λ_1 ⇒ collision with electrons (θ = 0) strongly bound to atom (electron not ejected)

→ collision between photon and atom

HW
3.34, 37, 38, 39

Exercise (x rays from TV)

e^- in TV: K = 25 keV

λ_{min} = ? $\quad h\nu = K - K' \Rightarrow \frac{hc}{\lambda_{min}} = K \Rightarrow \lambda_{min} = \frac{1240\ eV\cdot nm}{25\times10^3\ eV} = \boxed{0.050\,nm}$

(TV with protection against it)

Exercise In Compton scattering we find that λ_1 is shifted by 1.5 percent when θ = 120°

a) λ_1 = ? $\quad$ b) what is λ_2 when θ = 75°

a) $\Delta\lambda = 1.5\%\ \lambda_1$

$\lambda_2 - \lambda_1 = 0.015\lambda_1 = \frac{h}{mc}(1-\cos 120) \Rightarrow \lambda_1 = \frac{0.00243\ (1-\cos 120)}{0.015} = \boxed{0.243\,nm}$

b) $\lambda_2 - 0.243 = \frac{h}{mc}(1-\cos 75) \Rightarrow \boxed{\lambda_2 = 0.245\,nm}$

<u>low energies</u>	<u>medium energies</u>	<u>high energies</u>
UV	x-ray	very short x-rays γ-rays
small ν, large λ		
$\lambda \sim 10^{-8}$ m	$\lambda \sim 10^{-10}$ m	$\lambda \sim 10^{-12}$ m
<u>photoelectric effect</u>	<u>Compton effect</u>	<u>pair production</u>
(photon absorbed and e^- released)	(photon scattered)	(photon loses all its energy)

↓

photon collides with nucleus, loses all $h\nu$, creating one e^- (electron) and one e^+ (positron)

interaction of photons with matter

conversion of energy (photon) into rest mass (e^-, e^+) and K ←

(cosmic rays or particle accelerators)

Proton and anti-proton can also be created for higher energy photons.

∘) in interaction with matter:

radiation / light → localized particles / photons

∘) when propagating

radiation / light → wave (diffraction, interference)

<u>wave - particle DUALITY</u> } also extends to other particles: e^-, atoms, etc

(Louis de Broglie)

Classical physics fails when ν is large (λ is short) : blackbody, Compton effect

↳ relates to the size of h

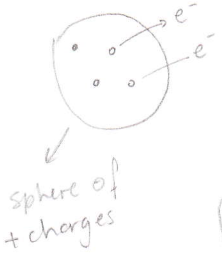
$$\begin{cases} h\nu \text{ small } (\nu \to 0) \Rightarrow \text{indistinguishable from continuum} \\ h\nu \text{ large } (\nu \to \infty) \Rightarrow \text{quantum effects cannot be neglected} \end{cases}$$

Nuclear Model

1910 - Experiments (scattering of x rays by atoms, photoelectric effect, etc) had been shown that atoms contain e^-. Since atoms are neutral, they must also contain positive charges. Because m_{e^-} is so small, $\Rightarrow$ most of the atom mass must be related to the positive charges

Thomson's Model : e^- were located within a continuous distribution of + charge

+ charge distribution – spherical, $r \sim 10^{-10}$ m (radius of the atom)



"plum pudding" model : e^- distributed through the sphere

Problem 1 :

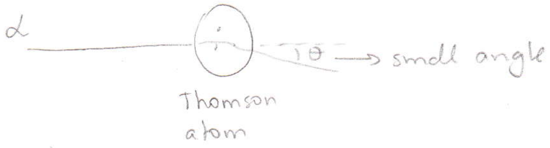
At lowest energy : e^- would be fixed at equilibrium position according to electrostatic forces → no possible configuration found

In excited atom : e^- would vibrate

↳ continuous emission of radiation

BUT not observed, and loss of energy – e^- would spiral into the nucleus

Problem 2 : could not explain large scattering angles of α particles



→) ⊕ charge distributed cannot provide intense Coulomb repulsion to explain large deflection (even $\theta = 180°$ is seen)

→) e^- are too small ⇒ only small deflection

Rutherford's Model

↳ ⊕ charge is not spread throughout atom but is concentrated in a small region at the center

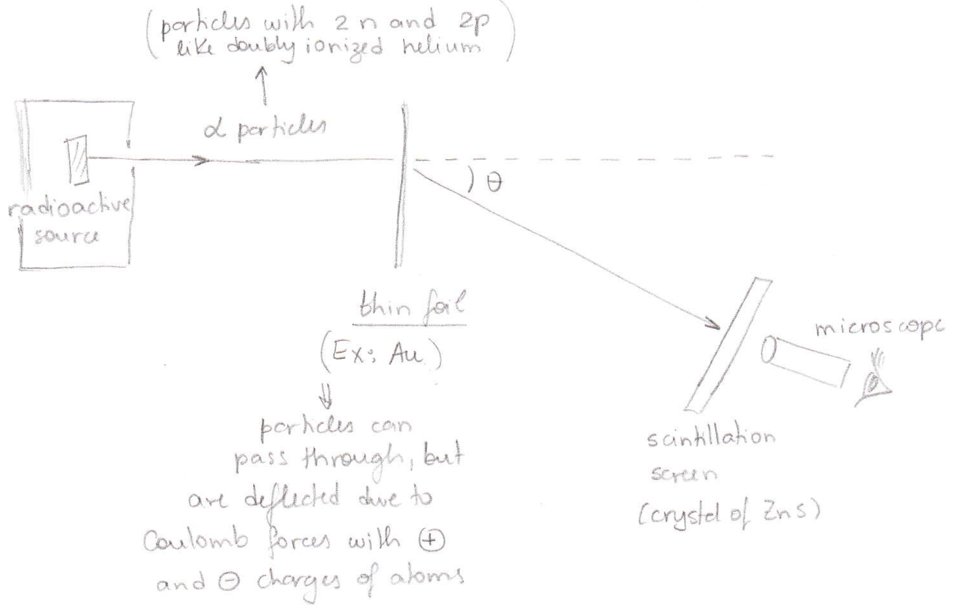
NUCLEUS → from 10^{-15} m to 10^{-14} m (1 fm – 10 fm)

1 fermi
femtometer } 1 fm = 10^{-15} m

solve Problem 2, but not Problem 1

↳ need Bohr's model

The experiment



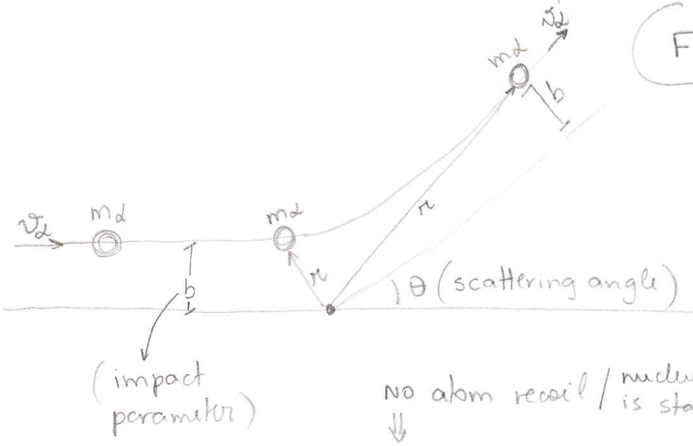
particles can pass through, but are deflected due to Coulomb forces with ⊕ and ⊖ charges of atoms

Rutherford's assumptions to obtain the theorical angular distribution for the scattered α particles (confirmed by his experiments):

For large angles:
- scattering due to repulsive Coulomb between α and ⊕ charged nucleus
- m of nucleus is so large compared to α ⇒ no recoil (heavy atom)
  - ↳ nucleus is a point charge / fixed
- α do not penetrate nuclear region
  - ↳ nucleus and α act as point charges

⇓

$$F = K \frac{q_\alpha Q}{r^2}$$



$$\boxed{b = \frac{K q_\alpha Q}{m_\alpha v^2} \cot(\theta/2)}$$

⇓

smaller b ⇒ larger θ

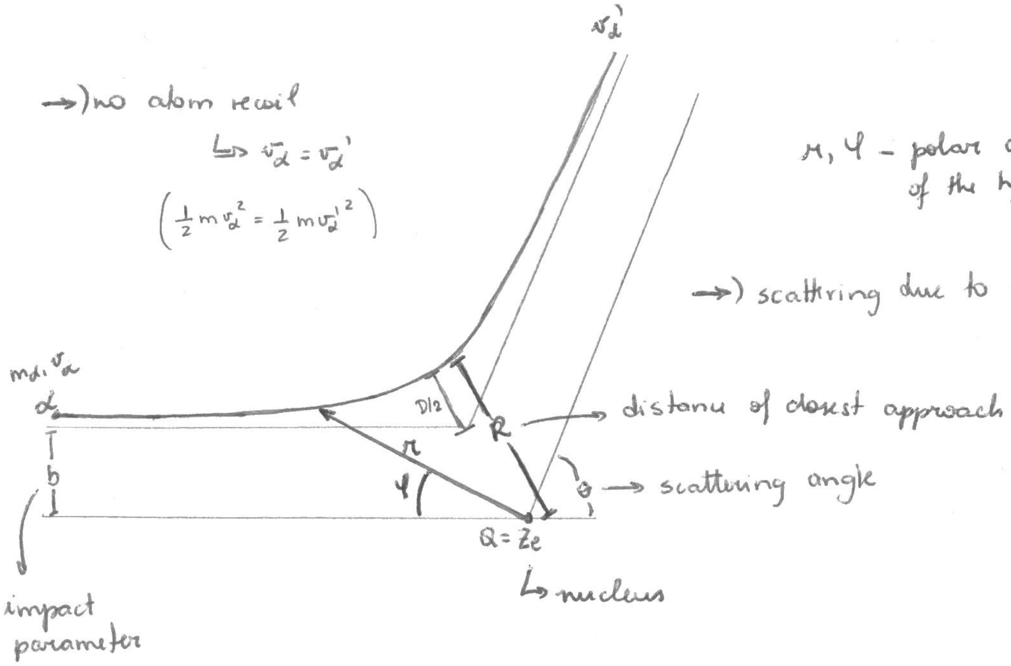
No atom recoil / nucleus is stationary

⇓

$v_\alpha = v_\alpha'$

$\frac{1}{2} m v_\alpha^2 = \frac{1}{2} m v_\alpha'^2$

The hyperbolic Rutherford trajectory



→) no atom recoil

↳ $v_\alpha = v_\alpha'$

$\left(\frac{1}{2} m v_\alpha^2 = \frac{1}{2} m v_\alpha'^2\right)$

r, φ – polar coordinates of the hyperbola

→) scattering due to repulsive Coulomb $F = \frac{k q_\alpha Q}{r^2}$

$$\boxed{\frac{1}{r} = \frac{1}{b}\sin\varphi + \frac{D}{2b^2}(\cos\varphi - 1)}$$ ← trajectory of the α particle

$$\boxed{D \equiv \frac{k q_\alpha Q}{\frac{1}{2} m_\alpha v^2}}$$ ← distance of closest approach in a head-on collision (b=0)

$$\frac{1}{2} m_\alpha v_\alpha^2 = \frac{k q_\alpha Q}{D}$$ ⇐ { distance where particle stops and reverses its direction of motion

initial kinetic energy = potential energy

→) For b>0 : particle does not stop, distance of closest approach R is larger than D

→) The scattering angle θ corresponds to π-φ as r→∞

$$\theta = \pi - \varphi$$

{ cos (a+b) = cos a cos b − sin a sin b

sin (a+b) = sin a cos b + sin b cos a }

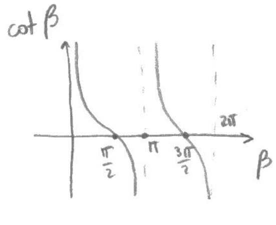
$$r \to \infty \Rightarrow \frac{1}{r} \to 0 \Rightarrow -\frac{1}{b}\sin(\pi-\theta) = \frac{D}{2b^2}\left(\cos(\pi-\theta) - 1\right)$$

$$-\frac{1}{\cancel{b}}\sin\theta = \frac{D}{2b^{\cancel{2}}}\left(-\cos\theta - 1\right)$$

$$b = (D/2)\,\frac{(2\cos^2\theta/2 - 1 + 1)}{2\sin\theta/2\,\cos\theta/2}$$

HW → show all steps

$$\boxed{b = \frac{D}{2}\cot(\theta/2)}$$



$0 \leq \theta \leq \pi$

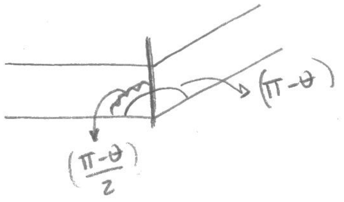
$\theta = 0 \Rightarrow b \to \infty$

$\theta = \pi \Rightarrow b \to 0$

HW
show all the steps

Exercise : Evaluate R, the distance of closest approach of the particle to the center of the nucleus

$r = R$ when $\varphi = (\pi - \theta)/2$



$$\frac{1}{R} = \frac{1}{b}\sin\left(\frac{\pi-\theta}{2}\right) + \frac{D}{2b^2}\left[\cos\left(\frac{\pi-\theta}{2}\right) - 1\right]$$

$$b = (D/2)\cot(\theta/2) = (D/2)\tan\left(\frac{\pi-\theta}{2}\right)$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan\left(\frac{\pi}{2} - x\right) = \frac{\sin(\pi/2 - x)}{\cos(\pi/2 - x)} = \frac{\cos x}{\sin x} = \cot x$$

⋮

$$\boxed{R = \frac{D}{2}\left[1 + \frac{1}{\sin(\theta/2)}\right]}$$

- As $\theta \to \pi$ (b = 0, head-on collision)

  $\hookrightarrow \boxed{R \to D}$

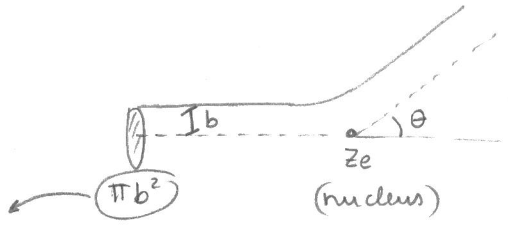
- As $\theta \to 0$ ($b \to \infty$, no deflection)

  $\hookrightarrow \boxed{R \to \infty}$

But b is not known in advance. Experiment measures number of detected α particles and θ.

→) I_0 : particles per second per unit area (intensity of incident α particle beam)

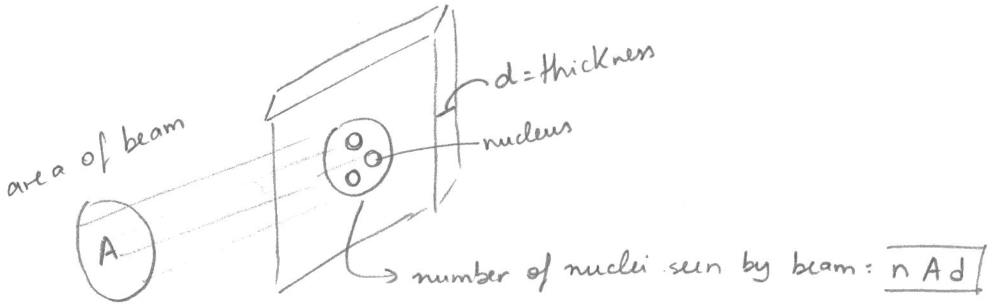
→) $(\pi b^2) I_0$: number of particles scattered per second by one nucleus through angles $\geq \theta$



cross section

for scattering through angles greater than θ

→) $(\pi b^2) I_0 \, n \, d A$: total number of particles scattered with angles greater than θ per second for a beam of area A through a foil of thickness d containing n nuclei per unit volume



$$n = \frac{N}{V}$$

$$\left.\begin{array}{l} N \longrightarrow M \\ N_A \longrightarrow M(1\,\text{mol}) \end{array}\right\} \quad N = \frac{M \, N_A}{M(1\,\text{mol})}$$

$$n = \frac{M}{V} \, \frac{N_A}{M(1\,\text{mol})} = \boxed{\rho \, \frac{N_A}{M(1\,\text{mol})}}$$

$$f = \frac{(\pi b^2) I_0 \, n \, d A}{I_0 A} = \text{fraction of the number of } \alpha \text{ particles scattered through angles greater than } \theta$$

$I_0 A$ → number of α particles incident per second

Experimentally we can mesure f and verify if it agrees with the theory

$$\boxed{f = \pi b^2 n d}$$

$$\boxed{n = \frac{\rho N_A}{M(1mol)}}$$

$$\boxed{b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot(\theta/2)}$$

For example:

for an incident beam of α particles with Kinetic energy 5 MeV, the fraction f of scattered particles from a gold foil with $\theta \geq 90°$ was verified to be

$\rho_{Au} = 19.3 \, g/cm^3$

(Z = 79)

10^{-6} m thick

$\boxed{f \sim 10^{-4}}$

↳ which agrees with the theorectical calculation as shown below

∘) $$n = \frac{\rho N_A}{M(1mol)} = \frac{(19.3 \, g/cm^3)(6.02 \times 10^{23} \, atoms/mol)}{(197 \, g/mol)} = 5.9 \times 10^{28} \, atoms/m^3$$

∘) $$b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot(\theta/2) = \frac{k(2e)(79e)}{2(5 \times 10^6 \, eV)} \overbrace{\cot(45°)}^{1} = 2.28 \times 10^{-14} \, m$$

$ke^2 = 1.440 \, eV \cdot nm$

$$\Rightarrow f = \pi (2.28 \times 10^{-14} \, m)^2 (5.9 \times 10^{28} \, atoms/m^3)(10^{-6} \, m)$$

$$f = 9.6 \times 10^{-5} \simeq \underline{10^{-4}}$$

Rutherford also derived an expression for the number of α particles ΔN scattered at any angle θ:

$$\boxed{\Delta N = \left(\frac{I_0 \, A_{sc} \, n d}{r_{fd}^2}\right)\left(\frac{k Z e^2}{2 E_k}\right) \frac{1}{\sin^4(\theta/2)}}$$

↳ Kinetic energy of incident particles

geometry of the detector

foil - detector distance

Exercise: α particles from ^{226}Ra are scattered at $\theta = 45°$ from a silver foil and 450 particles are counted each minute at the detector.

If the detector is moved to observe particles at 90°, how many will be counted per minute?

45° ⇒ $\frac{\Delta N}{min} = 450 = Const\ \sin^{-4}(45°/2) \Rightarrow Const = 450 \sin^{4}(45°/2)$

90° ⇒ $\frac{\Delta N}{min} = \left(450 \sin^{4}(45°/2)\right) \sin^{-4}(90°/2) \simeq 39$ particles/min

↳ Obs: (for very high energy, particles penetrate nucleus ⇒ $F = k\frac{q_\alpha Q}{r^2}$ is no longer valid and data don't agree with expression for ΔN)

Size of the nucleus

For 180° the collision is nearly 'head-on'
the closest approach of α to the nucleus is an experimental upper limit on the size of the nucleus.

Conservation of energy: $\underbrace{(V+E)_{\text{large } r}}_{\frac{1}{2} m_\alpha v^2} = \underbrace{(V+E)_{r_{nucleus}}}_{\frac{k q_\alpha Q}{r_d}}$

$$\boxed{r_d = \frac{k q_\alpha Q}{\frac{1}{2} m_\alpha v^2}} \sim 3\times10^{-14}\,\text{m for 7.7 MeV } \alpha \text{ particles}$$

Exercise:

$r_{Au} = 6.6$ fm. What K_α is necessary for this measurement?

$$K_\alpha = \frac{k q_\alpha Q}{r_{Au}} = \frac{(9\times10^{9}\ Nm^2/c^2)(2)(79)(1.60\times10^{-19}\,C)^2}{6.6\times10^{-15}\,m}$$

$$K_\alpha = 5.52\times10^{-12}\,J = \boxed{34.5\ MeV}$$

Stability of the Nuclear Atom

Rutherford's model ⇒ at the center of the atom there is a nucleus whose $m_{nucleus} \sim m_{atom}$ and $q_{nucleus} = Ze$

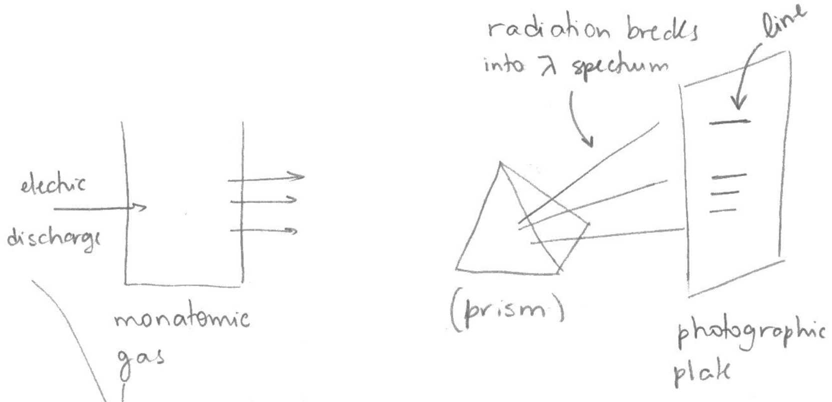
around nucleus : $Z\ e^-$

but stability ?
- stationary e^- ⇒ no possible stable arrangement
  e^- would fall into nucleus
- circulating as in the solar system ⇒ e^- would lose energy (accelerated e^- ⇒ radiation) and fall into nucleus

ALSO continuous spectrum that would be emitted in the process is not observed, spectrum is discrete

Solution: Bohr's model

Atomic Spectra



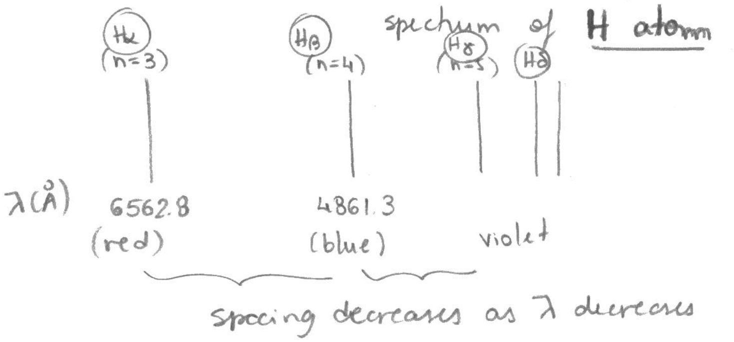
collisions ⇒ some atoms have more energy. When returning to normal energy state → give up excess energy by emitting electromagnetic radiation

electromagnetic radiation emitted by free atoms is concentrated at a number of discrete wavelengths

Each kind of atom has its own characteristic spectrum

SPECTROSCOPY

(1885) Balmer — access to **VISIBLE** spectrum of H atom



$$\boxed{\lambda = 3646\ \frac{n^2}{n^2-4}} \Leftarrow$$

1890 Rydberg: → extended to other elements

Balmer formula written for $1/\lambda$ for H →

$$K = \frac{1}{\lambda} = \boxed{R_H}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad n = 3, 4, 5$$

$\downarrow$ Rydberg constant = $10967757.6 \pm 1.2\, m^{-1}$

See Table 4-1, Eisberg, for Hydrogen series for UV and IR

spectroscopy

stars

Spectrum of free atoms → discrete

Bohr's postulate

↳ explains stability at lines of the spectrum of atoms

1) e^- moves in circular orbit around nucleus due to Coulomb attraction obeying laws of classical mechanics

2) only possible orbits: those for which L is an integral multiple of $\hbar$

$\hbar = h/2\pi$ $\quad (L = n\hbar)$ ← quantum number

3) e^- are constantly accelerating, but when moving in allowed orbit they do not radiate, E is const (stationary states)

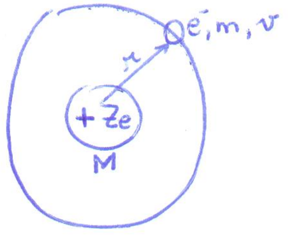
4) Electromagnetic radiation is emitted only when an e^- moves from one allowed orbit to another one. The frequency of the emitted radiation is

$$\nu = \frac{E_i - E_f}{h}$$

Bohr's Model

-) predictions derived from postulates agree with experiments

consider a single e^-



$Z = 1$ (hydrogen) - neutral
$Z = 2$ (singly ionized helium)
$Z = 3$ (doubly ionized lithium)

→) Assume:

- circular orbit
- $M >> m \Rightarrow$ nucleus is fixed

$$\circledast \quad \underbrace{\frac{kZe^2}{r^2}}_{\text{Coulomb}} = \underbrace{m\frac{v^2}{r}}_{ma}$$

↳ centripetal acceleration

→) Orbital angular momentum of e: } $\overline{L = mvr}$

quantization condition $\overline{L = n\hbar}$ ⇒ { only some radii exist / only some speeds exist

$mvr = n\hbar$ $\quad n = 1, 2, 3 \ldots$

↳ $v = \frac{n\hbar}{mr}$

→) ⊛ Find RADIUS: $r = \frac{kZe^2}{mv^2} \Rightarrow \cancel{r} = \frac{kZe^2}{\cancel{m}\, n^2\hbar^2} m^{\cancel{2}} r^{\cancel{2}}$

$$\boxed{r = \frac{n^2\hbar^2}{kmZe^2}}$$

$n = 1, 2, 3 \ldots$

↑ e^- has minimum energy

$> Z, < r$

$> n, > r$

$$a_0 = \frac{\hbar^2}{mke^2}$$ (Bohr radius)

$= 0.529$ Å ← Hydrogen atom in the ground state

→) Find speed of e^-: $\boxed{v = \frac{kZe^2}{nh}}$ $\quad n = 1, 2, 3 \ldots$

⇒ n cannot be zero

$> Z, \Rightarrow > v$

$> n \Rightarrow < v$

Coulomb

from $\frac{kZe^2}{r^2} = m\boxed{\frac{v^2}{r}}$ centripetal a

$$K = \frac{1}{2}mv^2 = \frac{kZe^2}{2r}$$

→) **TOTAL** energy of e^-

$$U = \ominus \frac{kZe^2}{r} \xleftarrow{\text{from}} U = -\int_r^\infty \frac{kZe^2}{r^2}dr = \ominus\frac{kZe^2}{r}$$

since Coulomb force is attractive, we need to do work to move e^- from r to ∞ against this force

$$E = K + U = -\frac{kZe^2}{2r}$$

$$r = \frac{n^2\hbar^2}{kmZe^2}$$

need energy to remove e^-

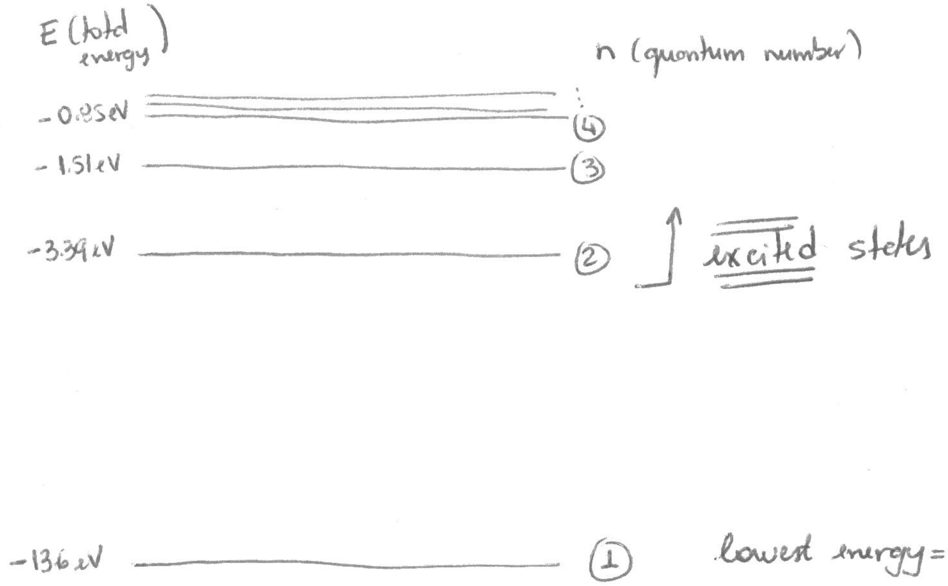
$$\boxed{E_0 = \frac{k^2me^4}{2\hbar^2}}$$

$$\boxed{E = \ominus \frac{k^2m(Ze^2)^2}{2\hbar^2}\frac{1}{n^2}}$$

$n = 1, 2, 3, \ldots$

$$\boxed{= -\frac{Z^2E_0}{n^2}}$$

↳ quantization of L leads to quantization of total energy of e^-



lowest energy = ground state ⤻ most stable state

most stable state, most negative

need the largest energy to remove e^-

→ E_0 { the binding energy of H is − 13.6eV
or ionization energy of H is ⊕ 13.6eV

→) **Frequency** of radiation emitted when e^- makes a transition from state n_i to n_f

higher energy, lower energy

$$\nu = \frac{E_i - E_f}{h} = \ominus\frac{k^2m(Ze^2)^2}{ⓗ\,2\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = \oplus\boxed{\frac{k^2m(Ze^2)^2}{2ⓗ\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

↳ ħ 2π

Electron makes a transition from state n_i to n_f

$$\nu = \frac{E_i - E_f}{h} = \boxed{\frac{E_0 Z^2}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$K = 1/\lambda = \nu/c$

$$K = \frac{1}{\lambda} = \frac{E_0 Z^2}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

↳ $\underbrace{\text{Bohr's prediction}}_{R_\infty}$ for the $\underbrace{\text{Rydberg}}_{R_H}$ constant

$n_f = 1$ Lyman series

$n_f = 2$ Balmer series : $K = R_\infty\left(1/2^2 - 1/n^2\right)$

⋮

→) Correction for Finite Nuclear Mass (Bohr's assumption that nucleus is fixed ↳ M is infinite)

↳ substitute m by μ. μ = reduced mass

$$\text{TOTAL } K = \underbrace{\frac{p^2}{2M}}_{\text{nucleus}} + \underbrace{\frac{p^2}{2m}}_{\text{electron}} = \frac{M+m}{2mM}\,p^2 = \frac{p^2}{2\mu}$$

$P_{nucleus} = P_{electron}$ $\underbrace{\text{(atom at rest)}}_{P_{atom}=0}$

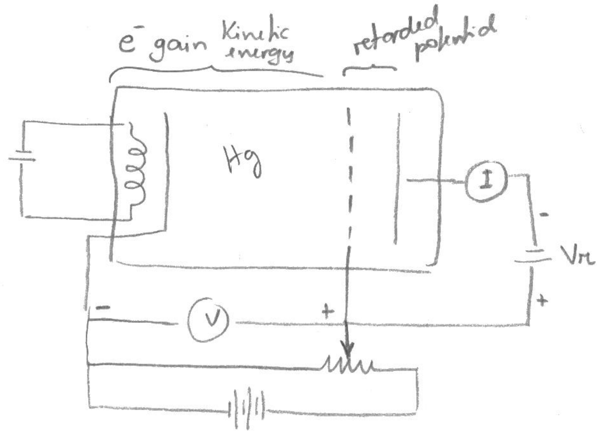
$$\boxed{\mu = \frac{mM}{m+M}}$$

new part

$$R = \frac{k^2 \mu e^4}{4\pi c \hbar^3} = R_\infty\left(\frac{1}{1 + m/M}\right)$$

Franck-Hertz experiment (1914)

↳ confirmation that the internal energy states of an atom are quantized.



at ~ 4.9 eV I drops

9.8 eV – again

etc

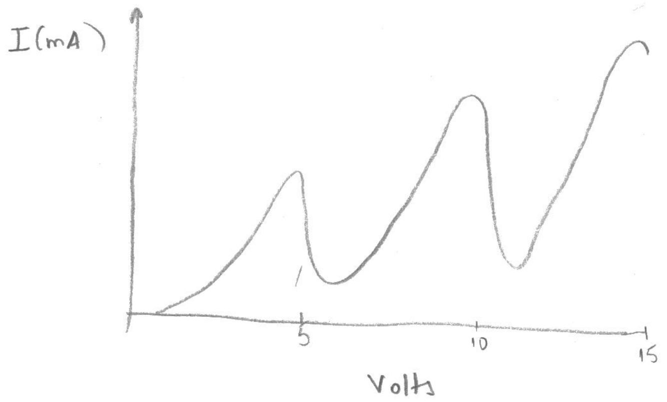
from ground state to 1st excited state

4.9 eV → e^- has energy to excite Hg atom

loses energy, cannot get enough to overcome Vr

9.8 eV → e^- has enough energy to excite ② Hg atoms

⋮



∘) Observing spectrum of Hg, Franck and Hertz saw emission line at 2536 Å

$$E = h\nu = \frac{hc}{\lambda} \xrightarrow{2536\text{Å}} E = \underline{\underline{4.9\,eV}} \quad (!!)$$

∘) e^- may also gain enough energy to excite Hg atom from ground to $\underbrace{\text{2nd}}_{6.7eV}$, 3rd ... excited state → drops in I also verified

→) can *directly* measure the energy differences of the quantum states of the atom

but

→) to find E of ground state, we need to ionize the atom

(DISCRETE SPECTRA is due to DISCRETE energy levels of the atom)

Old quantum theory

↳ improvements with respect to orbit → could be elliptical

↳ good predictions to several experiments

but it has limitations

- •) fails for atoms with more e^-
- •) cannot predict the rate of transitions $E_i \leftrightarrow E_f$

↳ is conceptually different from what we now call <u>quantum mechanics</u>

↓

(Schrödinger equation)

the idea of a
precise orbit does not make sense anymore

Δx — there is always an uncertainty in x

What remains from old quantum theory:

- •) $\frac{E_i - E_f}{h} = \nu$
- •) L can be quantized
- •) <u>CORRESPONDENCE PRINCIPLE</u>

(large temperatures)

"In the limit of large quantum numbers, large energies, quantum calculations must agree with classical calculations"

↓ which have worked so well in the macroscopic world

$$\text{classical limit} \begin{cases} n \to \infty \\ E \to \infty \\ \hbar \to 0 \end{cases}$$

HW
4.34, 4.36, 4.44
(Tipler)

symmetry
world is made of matter and radiation

Wave like properties of particles

Louis de Broglie

wave-particle dualism applies also to matter

photon has light wave associated with it that governs its motion

material particle ⟶ matter wave "

To describe particle, we need $\underbrace{E, p}$

↳ connect them wave concepts through h

$$\begin{cases} E = h\nu \\ p = h/\lambda \leftarrow \quad \boxed{\lambda = h/p} \end{cases}$$ (de Broglie relation)

↓ wave length of matter wave

Example 3.1 (Eisberg) a) What is the de Broglie wavelength of a baseball (1.0 Kg) moving at speed $v = 10 m/s$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6\times10^{-34}\,J.s}{1\,Kg\times10\,m/s} = \overline{\underline{6.6\times10^{-35}\,m}} = 6.6\times10^{-25}\,\mathring{A}$$

b) ... of an e^- where $K = 100 eV$?

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}\,J.s}{\left(2\times9.1\times10^{-31}Kg\times100\,eV\times1.6\times10^{-19}\frac{J}{eV}\right)^{1/2}} = \overline{\underline{1.2\times10^{-10}\,m}} = 1.2\,\mathring{A}$$

$$K = \frac{p^2}{2m}$$

o) visible light : $\underline{10^{-6} - 10^{-7} m}$

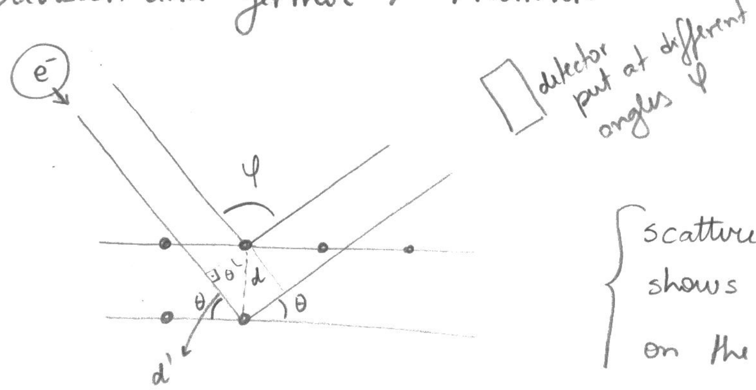
a: dimension of apparatus, slit

in optics if $a \gg \lambda \rightarrow$ geometrical optics : no $\underbrace{\text{diffraction, interference}}$

to observe wavelike aspects we need $\lambda \gtrsim a$

At de Broglie time, they could find $a \sim 1 Å$ between atoms in a crystal

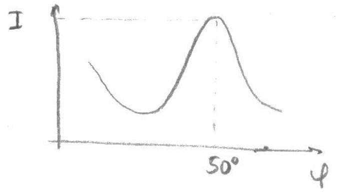
Davisson and Germer / Thomson ← Thomson's son



scattered electron beam shows a peak depending on the energy of e^-

For $V = 54V$ ⟶ peak at $\varphi = 50°$

↳ can only be explained as a constructive interference of waves (like Bragg diffraction)



$$\sin\theta = \frac{d'}{d} \Rightarrow \left\{ \begin{array}{l} \Delta = 2d' = 2d\sin\theta \\ \Delta = n\lambda \end{array} \right\} \Rightarrow \underline{n\lambda = 2d\sin\theta}$$

$$\varphi + 2\theta = \pi$$

$$\underbrace{\lambda = 2d\sin\left(\frac{\pi - \varphi}{2}\right) \qquad \text{crystal of nickel: } d = 0.91 Å}$$ (know from X-ray scattering)

⟶ $\lambda = 2 \times 0.91 \sin(65°) = \boxed{1.65 Å}$

⟶ $\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \ J.s}{\sqrt{2mK}} = \boxed{1.65 Å}$ (!)

$\uparrow$ $54 eV = 54 \times 1.6 \times 10^{-19} J$

Wave-particle duality

↳ each experiment can only see one aspect

↳ in a given measurement only one model applies

•) entity detected by some interaction with matter } acts like particle, is localized

•) when it moves / propagates: behaves like wave, interference, not localized

Principle of complimentarity : wave and particle models are complimentary (Bohr)

—— " ——

•) To describe { light waves → guide photons / radiation :

↳ wave for electric field

$$\mathcal{E}(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right]$$

→) satisfies wave equation : $\boxed{\frac{\partial^2 \mathcal{E}}{\partial x^2} = \frac{1}{(\lambda\nu)^2}\frac{\partial^2 \mathcal{E}}{\partial t^2}}$

(intensity) $I = \overline{\mathcal{E}^2}$

$I = \text{(N)} h\nu$ → average number of photons per unit time crossing unit area perpendicular to direction of propagation

$\overline{\mathcal{E}}^2 \leftrightarrow N$

↳ related to the probability of finding a photon in a unit area

•) To describe matter waves → guide material particles

$$\Psi(x,t) = A\sin\left[2\pi\left[\frac{x}{\lambda} - \nu t\right)\right]$$

wave function

satisfies Schrödinger equation

Ψ^2 probability of finding a particle in a unit volume at the position x at time t

1D : ⓧ

$\Psi(x,t)$ → wave function (complex)
$|\Psi(x,t)|^2$ → probability of finding particle at point x at time t

$$\int_a^b |\Psi(x,t)|^2\,dx = \begin{cases}\text{probability of finding the particle} \\ \text{between a and b at time t}\end{cases}$$

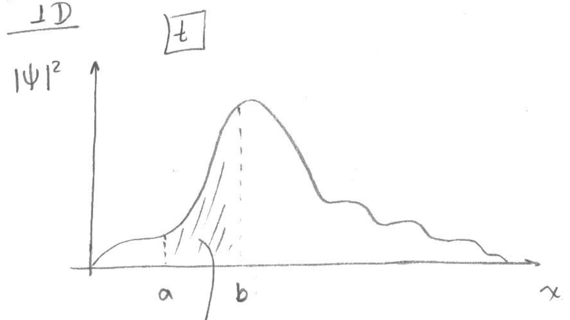
Contrary to classical physics, quantum physics is INTRINSICALLY probabilistic

- Classical physics: if we know the precise initial position AND momentum of a particle, we can predict its motion

- Quantum physics: can never know position AND momentum at the same time more accurately than is allowed by the Heisenberg uncertainty principle

$$\boxed{\Delta p\, \Delta x \geq \hbar/2}$$

1D



AREA
probability to find particle at t between a and b

To describe $\left\{ \begin{array}{l} \text{a } \underline{\text{localized particle}} \\ \text{a } \underline{\text{pulse}} \end{array} \right.$, just $\psi(x,t) = A \begin{array}{c}\cos\\ \sin\end{array} (kx - \omega t)$ is

not enough, we need a $\boxed{\text{wave packet}}$

$\boxed{\text{infinity}} \rightarrow$

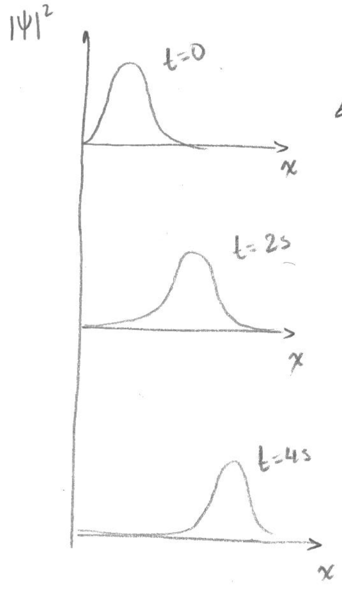
$$\psi(x,t) = \sum_{n=1}^{\infty} A_n \cos(k_n x - \omega_n t)$$

A → amplitude

$k = \frac{2\pi}{\lambda}$ → wave number

$\omega = 2\pi\nu$ → angular frequency

matter wave is in the form of a <u>GROUP</u>



<u>Mathematica</u>

that's what they tried with <u>Mathematica</u>

2 sin's ⇒ enveloppe moving slower

many sin's ⇒ more localized, but other peaks still present. For just one $\boxed{\int}$ instead of $\boxed{\Sigma}$

so waves won't be in <u>phase</u> anymore

→) Let us derive the uncertainty principle relation by combining

$p = h/\lambda$, $E = h\nu$ and properties of waves

Let us start with the sum of $\boxed{\text{two}}$ sinusoidal waves of slightly different λ and ν

$$\psi(x,t) = \sin(kx - \omega t) + \sin((k + dk)x - (\omega + d\omega)t)$$

$$\sin A + \sin B = 2\cos\left[(A-B)/2\right] \sin\left[(A+B)/2\right] = 2\left(\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}\right)\left(\sin\frac{A}{2}\cos\frac{B}{2} + \sin\frac{B}{2}\cos\frac{A}{2}\right)$$

$$= \sin A\cos^2\frac{B}{2} + \cos^2\frac{A}{2}\sin B + \sin^2\frac{A}{2}\sin B + \sin A\sin^2\frac{B}{2}$$

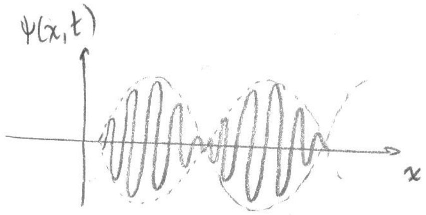
sin A   sin B

✓

$$\Psi(x,t) = 2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)\sin\left(\left(k+\frac{dk}{2}\right)x - \left(\omega+\frac{d\omega}{2}\right)t\right)$$

since $d\omega \ll \omega$ and $dk \ll k$

$$\boxed{\Psi(x,t) = \underbrace{2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)}_{\substack{\text{enveloppe,}\\ \text{it modulates } \sin(kx-\omega t)}}\sin(kx-\omega t)}$$



In mathematica - past HW

$p_1 = \sin[1.x]$

$p_2 = \sin[1.1x]$

$\begin{cases} A = ?,\ t = ? \\ dk = ? \end{cases}$

$pe = 2\cos[0.1/2\,x]$ ← what is it?

Suppose a particle moving at speed $\boxed{v}$. what is the velocity of the associated matter wave?

↳ The associated wave is the $\left\{\begin{matrix}\text{packet}\\ \text{GROUP}\end{matrix}\right\}$, which also moves with v

→) group velocity: given by the enveloppe

$$\boxed{v_g = \frac{d\omega}{dk}}$$

careful (!) $\begin{cases} k = 2\pi/\lambda \\ \dfrac{dk}{d\lambda} = -\dfrac{2\pi}{\lambda^2} \end{cases}$

$E = h\nu$ and $\omega = 2\pi\nu \Rightarrow \boxed{E = \hbar\omega}$

$p = h/\lambda$ and $k = 2\pi/\lambda \Rightarrow \boxed{p = \hbar k}$

$$\boxed{v_g = \frac{dE}{dp}}$$

$$v_g = \frac{dE}{dp} \overset{E = p^2/2m}{=} \frac{2p}{2m} = \boxed{v}$$

→) velocity of individual wave $\sin(kx-\omega t)$ is smaller

$$v_w = \lambda\nu = \frac{E}{p} = \frac{1/2\,\cancel{m}v^{\cancel{2}}}{\cancel{m}\cancel{v}} = \boxed{\frac{v}{2}}$$

Therefore, to describe a localized particle we use a wave packet with group velocity $v_g = \frac{d\omega}{dk}$ equal to the velocity of the particle

→ Sum of $\boxed{\text{more}}$ waves

$$\boxed{t=0}$$

$$\psi(x) = \begin{cases} 1/4 \cos[2\pi\, 9x] \\ + 1/3 \cos[2\pi\, 10x] \\ + 1/2 \cos[2\pi\, 11x] \\ + 1 \cos[2\pi\, 12x] \\ + 1/2 \cos[2\pi\, 13x] \\ + 1/3 \cos[2\pi\, 14x] \\ + 1/4 \cos[2\pi\, 15x] \end{cases}$$

$$\psi'(x) = \begin{cases} 1/4 \cos[9x] \\ + 1/3 \cos[10x] \\ + 1/2 \cos[11x] \\ + 1 \cos[12x] \\ + 1/2 \cos[13x] \\ + 1/3 \cos[14x] \\ + 1/4 \cos[15x] \end{cases}$$

Δx = width at half of the main peak

$$\begin{cases} \Delta x \approx 0.05 \\ \\ \Delta k \approx 4\pi \end{cases}$$

Δk = width at half of the peak for

$$\begin{cases} \Delta x \approx 0.2 \\ \\ \Delta k = 1 \end{cases}$$

<u>Conclusion</u>: larger $\Delta k \Rightarrow$ narrower Δx

Δx is inversely proportional to Δk

We need Fourier integral to prove that $\Delta x\, \Delta k \geq 1/2 \xrightarrow{p=\hbar k} \boxed{\Delta x\, \Delta p \geq \hbar/2}$

HW 5.4, 5.7, 5.10, 5.17 a b c
, 5.40 a b
5.42, 5.44
(Tipler)

SUMMARY

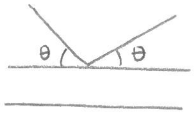
•) wave-particle duality also applies to matter

$$\begin{cases} E = h\nu \xrightarrow{\omega = 2\pi\nu} \boxed{E = \hbar\omega} \\ p = h/\lambda \xrightarrow{k = 2\pi/\lambda} \boxed{p = \hbar k} \end{cases}$$

•) to detect wave aspects: $\boxed{\lambda \gtrsim a}$ size of slit

double-slit experiment: interference

Davisson-Germer experiment: diffraction

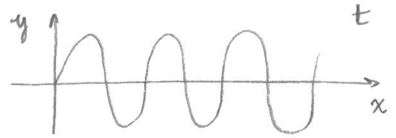


certain θ ⇒ peak in I (constructive interference)

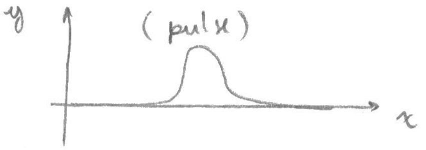
other values of θ' ⇒ NO I (destructive interference)

mathematical qualitative justification for

•) uncertainty principle



know $\boxed{\lambda}$, $x = ?$



know $\boxed{x}$, $\lambda = ?$

pulse → localized particle

$$y(x,t) = \sum A_n \cos(k_n x - \omega_n t)$$

the more spread k, p, λ ⇒ the more localized is the particle

(wave packet)

phase velocity $v_p = \frac{\omega}{k}$

group velocity (follows the particle) $\boxed{v_g = \frac{d\omega}{dk}}$

ⓣ $\Delta x \Delta k \sim 1$ (property of any wave) (!)

ⓧ $\Delta t \Delta \omega \sim 1$ (property of any wave) (!)

$\boxed{\Delta x \Delta p \geq \hbar/2}$ ← Quantum mechanics (!) → $\boxed{\Delta t \Delta E \geq \hbar/2}$

Δt: rate of change of an observable
Δt is small, quick change if uncertainty in E is large

Δx, Δp: uncertainty, spread

measurements on identically prepared systems DO NOT give identical results

The Uncertainty Principle

(mathematically: $\psi = \sum \cos \Rightarrow \Delta x \, \Delta p$
physically: measurement disturbs the system)

↳ explains principle of complimentarity:

If experiment forces $\begin{bmatrix}\text{matter}\\ \text{radiation}\end{bmatrix}$ to reveal its wave $\overset{(\lambda)\to(p)}{\text{charecter}}$, it suppresses its particle $\underset{(x)}{\text{charecter}}$ (and vice-versa)

(Two-slit experiment)
Feymann's text

The precision of measurement is inherently limited by the measurement process itself such that

$$\boxed{\Delta p_x \Delta x \geq \hbar/2} \qquad (\hbar = h/2\pi)$$

Even with ideal instruments we can never do better than $\Delta p_x \Delta x \geq \hbar/2$

Restriction is not on accuracy to which x and p_x can be measured, but on the product $\Delta p_x \Delta x$ in a simultaneous measurement of both

NOTE: Planck's constant characterizes the quantum result / the limitation on measurements

If $\boxed{\hbar \to 0}$ (classical domain) ⇒ no limitations on measurements (classical view)
(correspondence principle)

In the microscopic domain, we cannot determine x and p simultaneously,
cannot specify initial conditions exactly
cannot precisely determine the future behavior of a system

↓

we can only talk about probabilities of events

A physical justification for the uncertainty principle

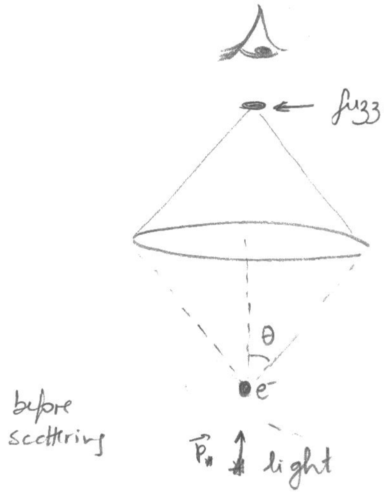
A measurement disturbs the system...

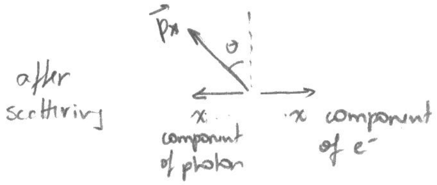
To measure the position of an e^-, we need light (microscope) → at least one photon

The photon is scattered by the e^- and comes back to our 'eyes'

⇒ Resolving power of a microscope: $\Delta x = \frac{\lambda}{2\sin\theta}$ → accuracy to which e^- can be located

•) To reduce Δx, need shorter λ





⇒ Photon initial momentum is p
after colliding with e^-, it is scattered within the angular range 2θ
so the x component of the photon varies from

$$-p\sin\theta \text{ to } p\sin\theta \Rightarrow \underbrace{\Delta p_x = 2p\sin\theta}_{\text{uncertainty after scattering}}$$

but the e^- receives a recoil momentum in x of the same magnitude

$$\Delta p_x = 2p\sin\theta = 2\frac{h}{\lambda}\sin\theta$$

$$\Delta x\,\Delta p = \frac{\lambda}{2\sin\theta}\,2\frac{h}{\lambda}\sin\theta = h \xrightarrow[\text{estimate for}]{\text{good}} \Delta x\,\Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

to reduce Δx needs shorter λ ⟷ to reduce Δp needs larger λ

consequence of uncertainty principle
↳ zero point energy / quantum fluctuations
particle cannot be at rest even at temperature = 0

Schrödinger equation

(1925)

↳ governs the propagation of matter waves
wave equation governing the motion of particles with mass

(We can't derive Schröd. eq., as we can't derive Newton's laws of motion
Its validity lies in its agreement with experiments)

⇒) Wave equation for photons

$$\boxed{\frac{\partial^2 \mathcal{E}}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2}}$$

photon moves at the speed of light ⓒ

$$\mathcal{E}(x,t) = \mathcal{E}_0 \cos(kx-\omega t)$$

$$\underbrace{\frac{\partial^2 \mathcal{E}}{\partial x^2} = -k^2 \mathcal{E} \qquad \frac{\partial^2 \mathcal{E}}{\partial t^2} = -\omega^2 \mathcal{E}}$$

$$k^2 = \frac{\omega^2}{c^2} \Rightarrow \boxed{\omega = kc}$$

using $\omega = E/\hbar$ and $p = \hbar k$ $\longrightarrow$ $\boxed{E = pc}$

relation between E and p for photon (relativistic particle)

⇒) Schrödinger eq applies to $\boxed{\text{nonrelativistic}}$ problems
(Dirac eq. to relativistic case)

Total energy of nonrelativistic particle. $E = \frac{p^2}{2m} + V$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$$

linear in ω
quadratic in k^2 ⇒

$\boxed{\neq \text{ from photon } \omega^2 = k^2c^2}$

⇒ Schröd. eq. relates ①st time derivative to the ②nd space derivative
and also involves the potential energy

The eq. also needs to be LINEAR in the wave function ⇒ ψ_1 is a solution, ψ_2 is a solution ⇒

⇒ any linear combination $\psi = a_1\psi_1 + a_2\psi_2$ is also a solution (interference can happen)

Schrödinger Equation in 1D

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

equation is <u>LINEAR</u> in $\Psi(x,t)$ ⇒ if Ψ_1 and Ψ_2 are two ≠ solutions for given V, any <u>arbitrary</u> <u>linear</u> <u>superposition</u> $a_1\Psi_1 + a_2\Psi_2$ is a solution

Example 5-2 (Eisberg)

$$\Psi = a_1\Psi_1 + a_2\Psi_2$$

$$a_1\underbrace{\left(-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1}{\partial x^2} + V\Psi_1 - i\hbar\frac{\partial\Psi_1}{\partial t}\right)}_{(=0)} =$$

$$-a_2\underbrace{\left(-\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2}{\partial x^2} + V\Psi_2 - i\hbar\frac{\partial\Psi_2}{\partial t}\right)}_{(=0)}$$

<u>free particle</u> : no net force acts on a free particle

↳ $V(x,t) = V_0 \qquad \left(F = \frac{\partial V(x,t)}{\partial x} = 0\right)$

just $\cos(kx-\omega t)$ or $\sin(kx-\omega t)$ <u>don't</u> satisfy the eq.

$$\Psi(x,t) = \underset{\text{constant}}{A}\, e^{i(kx-\omega t)} = A\left[\cos(kx-\omega t) + i\sin(kx-\omega t)\right]$$

$$\frac{\partial\Psi}{\partial t} = -i\omega\Psi$$

$$\frac{\partial\Psi}{\partial x} = ikA \rightarrow \frac{\partial^2\Psi}{\partial x^2} = -k^2\Psi$$

$$\frac{\hbar^2k^2}{2m}\Psi + V_0\Psi = \hbar\omega\Psi$$

↳ $\frac{\hbar^2k^2}{2m} + V_0 = \hbar\omega$

$$\left(\frac{p^2}{2m} + V_0 = E\right)$$

- The wave function $\Psi(x,t)$ may be <u>COMPLEX</u> → it is a <u>computational</u> device

- $P(x,t)\,dx$: <u>probability</u> to find particle in the volume dx

$$P(x,t)\,dx = \Psi^*(x,t)\,\Psi(x,t)\,dx = |\Psi(x,t)|^2\,dx$$

↳ has physical significance, is <u>REAL</u>

<u>Born's statistical</u> <u>interpretation</u>

(QM is intrinsically probabilistic)

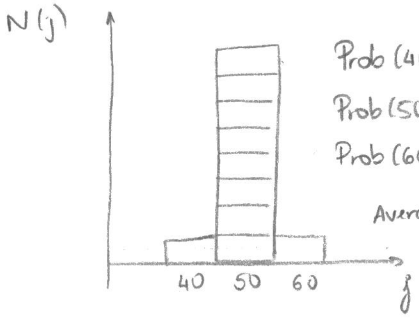
$P(x,t)$: probability density for finding particle at point x at time t

$\Psi(x,t)$: probability (density) amplitude

NOTES ON STATISTICS

in a room
$$\left\{\begin{array}{l}1 \text{ person: } 40 \text{ years old}\\ 8 \text{ persons: } 50\\ 1 \text{ person: } 60\end{array}\right.$$

total: $\boxed{10}$ // average: $\frac{40+8\times50+1\times60}{10} = \boxed{50}$ //



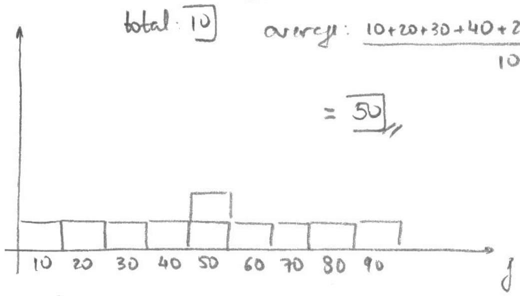
$$\left.\begin{array}{l}\text{Prob}(40) = 1/10\\ \text{Prob}(50) = 8/10\\ \text{Prob}(60) = 1/10\end{array}\right\} \boxed{\text{Prob}_{\text{total}} = 1} //$$

Average: $0.1\times40+0.8\times50+0.1\times60$

another room
$$\left\{\begin{array}{ll}1 \text{ person} : & 10\\ \text{''} : & 20\\ \text{''} : & 30\\ \text{''} : & 40\\ 2 \text{ persons} : & 50\\ 1 \text{ person} : & 60\\ 1 \text{ ''} : & 70\\ \text{''} & 80\\ \text{''} & 90\end{array}\right.$$

total: $\boxed{10}$ average: $\frac{10+20+30+40+2\times50+60+70+80+9}{10}$

$= \boxed{50}$ //



more spread

↳ larger $\left\{\begin{array}{l}\text{dispersion}\\ \text{standard deviation}\end{array}\right\}$ σ

variance

$$\sigma^2 \equiv \langle(\underbrace{j-\langle j\rangle}_{\text{how much individuals deviate from average } \langle j\rangle})^2\rangle$$

$$\sigma^2 = \langle j^2 - 2j\langle j\rangle - \langle j\rangle^2\rangle = \langle j^2\rangle - 2\langle j\rangle\langle j\rangle - \langle j\rangle^2 = \boxed{\langle j^2\rangle - \langle j\rangle^2}$$

Dispersion / standard deviation

$$\boxed{\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}} //$$

uncertainty in x and p

$$\left\{\begin{array}{l}\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}\\ \\ \Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}\end{array}\right.$$

- $\Psi(x,t)$: probability amplitude
- $\int_a^b |\Psi(x,t)|^2\,dx$: probability to find the particle between a and b at time t

$$\boxed{\int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx = 1} \leftarrow \text{normalization condition}$$

→) wave function contains information about the behavior of the associated particle

Expectation Values

→) the most we can know about a particle position is the probability that a measurement will yield a value x

$$\langle x\rangle = \int_{-\infty}^{\infty} x\, P(x)\,dx = \int_{-\infty}^{\infty} x\,|\Psi(x,t)|^2\,dx = \int_{-\infty}^{\infty} \Psi^*(x,t)\, x\, \Psi(x,t)\,dx$$

→) expectation value is the average of repeated measurements on a ensemble of identically prepared systems

(not the average of repeated measurements on one and the same system, because after a measurement we COLLAPSE the wave function)

$$\langle x^2\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\, x^2\, \Psi(x,t)\,dx$$

any function f(x)

$$\langle f(x)\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\, f(x)\, \Psi(x,t)\,dx$$

↳ how can we find $\langle p\rangle$? p is NOT function of x, because of the uncertainty princ.

- one way is to use the wave function for momentum $\Psi(p,t)$ [Fourier transform $\Psi(x,t) \rightarrow \Psi(p,t)$]
- the other way is to use $p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x}$

$$\begin{cases} \frac{p^2}{2m} + V_0 = E & \xleftarrow[\text{particle}]{\text{free}} \quad \Psi(x,t) = A e^{i(kx-\omega t)} \\ \\ \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V_0\Psi = i\hbar\frac{\partial\Psi}{\partial t} & \end{cases}$$

$$\hookrightarrow \boxed{p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x}} \qquad \underline{\text{momentum}} \; \underline{\text{operator}}$$

HW

5.24, Prob. 1.9 (Griffiths), Prob. 1.17 (Griffiths)

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\pi/\alpha}$$

$$\int_{-\infty}^{\infty} x\, e^{-\alpha x^2}\,dx = 0$$

$$\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\,dx = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$$

$$\langle p\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi(x,t)\,dx$$

$$\langle p^2\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi(x,t)\,dx$$

→ no forces act on the particle

Example 5-9 (Eisberg) Consider a particle of mass m which can move freely along the x axis anywhere from $x = -a/2$ to $x = a/2$, but which is strictly prohibited from being found outside this region. The particle bounces back and forth between the walls at $x = \pm a/2$ of a one-dimensional box.

The wavefunction for the lowest energy state of this particle is

$$\Psi(x,t) = \begin{cases} A\cos\frac{\pi x}{a}\, e^{-iEt/\hbar} & -a/2 < x < a/2 \\ \\ 0 & x \leq -a/2 \text{ or } x \geq a/2 \end{cases}$$

a) Show that $\Psi(x,t)$ is a solution to the Schrödinger eq. and determine E

b) Determine the value of the constant A so that the probability to find the particle somewhere in x is 1

c) Evaluate $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$

d) Show that $\Delta x\,\Delta p$ is consistent with the uncertainty principle

no forces act on the particle ⇒ V= const
↳ can choose V=0

a) $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \cancel{V\psi} = i\hbar\frac{\partial\psi}{\partial t}$$ $$\psi = A\cos\frac{\pi x}{a}\, e^{-iEt/\hbar}$$

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi$$

$$\frac{\partial\psi}{\partial x} = -\frac{\pi}{a}A\sin\frac{\pi x}{a}\, e^{-iEt/\hbar} \longrightarrow \frac{\partial^2\psi}{\partial x^2} = -\left(\frac{\pi}{a}\right)^2\psi$$

to satisfy Schröd eq ⇒ $\boxed{+\frac{\hbar^2}{2m}\frac{\pi^2}{a^2} = E}$

$\cos(2y) = \cos^2 y - \sin^2 y = 2\cos^2 y - 1$
↳ $\cos^2 y = \frac{1+\cos 2y}{2}$

b) $$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \Rightarrow \int_{-a/2}^{a/2} A^2\cos^2\left(\frac{\pi x}{a}\right)dx = 1$$

$$A^2\left[\underbrace{\int_{-a/2}^{a/2}\frac{dx}{2}}_{a/2} + \underbrace{\int_{-a/2}^{a/2}\frac{1}{2}\cos\left(\frac{2\pi x}{a}\right)dx}_{y=\frac{2\pi x}{a} \quad dx=\frac{a}{2\pi}dy}\right] = A^2\left[\frac{a}{2} - \frac{a}{4\pi}\cancel{\sin y\Big|_{-\pi}^{\pi}}\right] = A^2\frac{a}{2}$$

$$\frac{1}{2}\int_{-\pi}^{\pi}\frac{a}{2\pi}\cos y\, dy$$

↳ $\boxed{A = \sqrt{2/a}}$

c) $$\langle x\rangle = \int_{-a/2}^{a/2}\frac{2}{a}\overset{\text{even}}{\cos^2\frac{\pi x}{a}}\ \overset{\text{odd}}{x}\ dx = 0$$

EVEN function : $f(x) = f(-x)$
Examples: x^2, $\cos x$

ODD function : $-f(x) = f(-x)$
Examples : x, x^3, $\sin x$

$$\langle x^2\rangle = \frac{2}{a}\int_{-a/2}^{a/2}\cos^2\left(\frac{\pi x}{a}\right)x^2 dx = \frac{1}{a}\left[\int_{-a/2}^{a/2}x^2 dx + \int_{-a/2}^{a/2}\cos\left(\frac{2\pi x}{a}\right)x^2 dx\right]$$

$y = \frac{2\pi x}{a}$ $dx = \frac{a}{2\pi}dy$

$$\int_{-a/2}^{a/2} \cos\left(\frac{2\pi x}{a}\right) x^2\,dx = \frac{a}{2\pi}\,\frac{a^2}{4\pi^2}\overbrace{\underbrace{\int_{-\pi}^{\pi} \cos y \; y^2\,dy}}^{-4\pi}$$

$u = y^2$ $\quad dv = \cos y\,dy$

$du = 2y\,dy$ $\quad v = \sin y$

$$\cancel{y^2 \sin y\Big|_{-\pi}^{\pi}} - 2\underbrace{\int_{-\pi}^{\pi} y \sin y\,dy}$$

$u = y$ $\quad dv = \sin y\,dy$

$du = dy$ $\quad v = -\cos y$

$$\underbrace{-y\cos y\Big|_{-\pi}^{\pi}}_{+\pi + (\pi) = 2\pi} + \underbrace{\cancel{\int_{-\pi}^{\pi} \cos y\,dy}}_{\sin y\Big|_{-\pi}^{\pi}}$$

$$\langle x^2\rangle = \frac{1}{a}\left[\frac{(a/2)^3}{3} - \frac{(-a/2)^3}{3} - \frac{a^3}{2\cancel{8}\pi^{\cancel{3}2}}\cancel{4\pi}\right] = \frac{a^2}{12} - \frac{a^2}{2\pi^2} = \boxed{\frac{a^2}{2\pi^2}\left(\frac{\pi^2}{6} - 1\right)} = 0.033a^2$$

$$\langle p\rangle = \frac{2}{a}\int_{-a/2}^{a/2} \cos\frac{\pi x}{a}\underbrace{\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\cos\frac{\pi x}{a}\right)}_{-\frac{\hbar}{i}\frac{\pi}{a}\sin\frac{\pi x}{a}} dx = -\frac{2\hbar\pi}{i a^2}\int_{-a/2}^{a/2} \overset{\text{even}}{\cos\frac{\pi x}{a}}\;\overset{\text{odd}}{\sin\frac{\pi x}{a}}\,dx = 0$$

$$\langle p^2\rangle = \frac{2}{a}\int_{-a/2}^{a/2}(-\hbar^2)\cos\frac{\pi x}{a}\underbrace{\frac{\partial^2 \cos \pi x/a}{\partial x^2}}dx = \frac{2}{a}\hbar^2\frac{\pi^2}{a^2}\underbrace{\int_{-a/2}^{a/2}\cos^2\left(\frac{\pi x}{a}\right)dx}$$

$$-\frac{\pi}{a}\frac{\partial}{\partial x}\sin\frac{\pi x}{a}$$

$$-\left(\frac{\pi}{a}\right)^2\cos\frac{\pi x}{a}$$

$$\underbrace{\int_{-a/2}^{a/2}\frac{dx}{2}}_{\frac{a}{2}} + \underbrace{\int_{-a/2}^{a/2}\cos\left(\frac{2\pi x}{a}\right)dx}$$

$y = \frac{2\pi x}{a}$

$dx = \frac{a\,dy}{2\pi}$

$$\int_{-\pi}^{\pi}\cos y\,dy = \cancel{-\sin y\Big|_{-\pi}^{\pi}} = 0$$

$$\langle p^2\rangle = \frac{\cancel{2}\hbar^2\pi^2}{a^{\cancel{3}2}}\frac{\cancel{a}}{\cancel{2}} \longrightarrow \boxed{\langle p^2\rangle = \left(\frac{\hbar\pi}{a}\right)^2}$$

d) $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\langle x^2 \rangle} = 0.18\, a$

$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\langle p^2 \rangle} = \frac{\hbar \pi}{a}$

$\Delta x\, \Delta p = 0.18\, \cancel{a}\, \frac{\hbar \pi}{\cancel{a}} = \boxed{0.57\, \hbar}$ which is consistent with the lower
limit $\hbar/2$ set by the
uncertainty principle

Relativity

(Tipler: Chapters 1 and 2 , Eisberg: Appendix A)

Newton's first law: "every object continues in its state of rest, or uniform velocity in a straight line, as long as no net force acts on it"

(Law of Inertia)

In any inertial reference frame, a particle moves without any change in velocity if no net force acts on it

⊛ Special theory of relativity deals with inertial frames

↑ our subject

(Einstein - 1905)

↑

(Annus Mirabilis)

(frames of reference moving with constant velocity relative to each other)

→ General theory of relativity deals with accelerated frames and gravity

(Einstein - 1916)

(deformation of space-time caused by large masses)

subject of more advanced courses

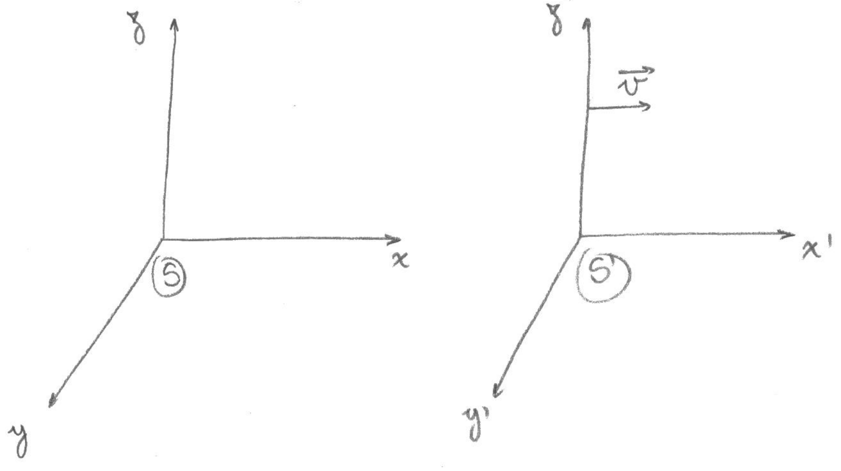
1) How do we transform our description of a system from one frame to another one? (coordinates)

2) What happens to the equations which govern the behavior of the system when we make the transformation?

Classical physics: to describe the state of a system we need a reference frame → use it for coordinates, time derivatives of coord

Given $\textcircled{m}$ and $\textcircled{\vec{F}}$; Newton's equations calculate the state at any future time

→) assume that the origins of S and S' coincide at $t = t' = 0$



$\overbrace{(x, y, z, t)}^{S}$ or $\overbrace{(x', y', z', t')}^{S'}$ can be equally used to specify the coordinates of the particle at any instant of time

Galilean Transformation

Answer to question ①

Particle in S' seen by S

$$\begin{cases} x = x' + vt \\ y = y' \\ z = z' \\ \boxed{t = t'} \end{cases} \Rightarrow \boxed{u_x = u_x' + v}$$

Particle in S seen by S'

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ \boxed{t' = t} \end{cases} \Rightarrow \boxed{u_x' = u_x - v}$$

Answer to question ②

$\underline{S}$ (Newton's 2nd law)

$$\begin{cases} m\dfrac{d^2x}{dt^2} = F_x \\ m\dfrac{d^2y}{dt^2} = F_y \\ m\dfrac{d^2z}{dt^2} \end{cases}$$

$$\left.\begin{matrix} \dfrac{dx'}{dt} = \dfrac{dx}{dt} - v \Rightarrow \dfrac{d^2x'}{dt^2} = \dfrac{d^2x}{dt^2} \\ \dfrac{dy'}{dt} = \dfrac{dy}{dt} \Rightarrow \dfrac{d^2y'}{dt^2} = \dfrac{d^2y}{dt^2} \\ \dfrac{dz'}{dt} = \dfrac{dz}{dt} \Rightarrow \underbrace{\dfrac{d^2z'}{dt^2} = \dfrac{d^2y}{dt^2}}_{\text{acceleration is the same is S and S'}} \end{matrix}\right\} \xRightarrow{\text{mass}}$$

is an intrinsic property of the particle

$$\begin{cases} \cancel{F_x' = F_x} \\ \cancel{F_y' = F_y} \\ \cancel{F_z' = F_z} \end{cases}$$

and $F_x' = F_x$, $F_y' = F_y$, $F_z' = F_z$ $\Longrightarrow$

$$F_x = m\frac{d^2x}{dt^2} = m\frac{d^2x'}{dt^2}$$ and since $F_x = F_{x'} \Rightarrow F_{x'} = m\frac{d^2x'}{dt^2}$

$$m\frac{d^2x'}{dt'^2} = F_{x'} \qquad m\frac{d^2y'}{dt'^2} = F_{y'} \qquad m\frac{d^2z'}{dt'^2} = F_{z'}$$

Newton's law of motion are INVARIANT
for any inertial frame
under a galilean transformation

BUT Maxwell's equations do change their mathematical form under a Galilean transformation

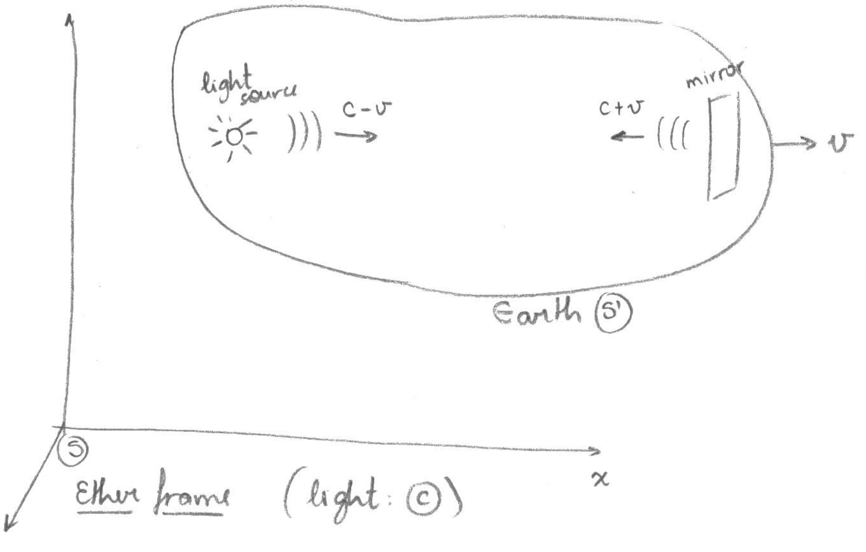
⟶ Maxwell's equations predicted the existence of electromagnetic waves whose speed was ⓒ

WRONG XIX century beliefs

- they must propagate in a medium called ether
- there was only one frame, the ether frame where the velocity of light was ⓒ

Michelson - Morley experiment

The motion of Earth through the ether should be detectable

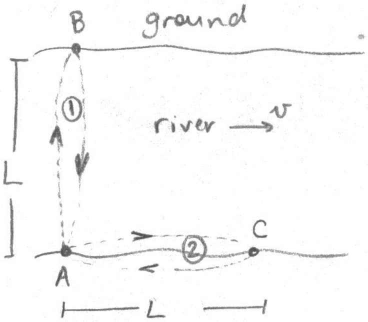


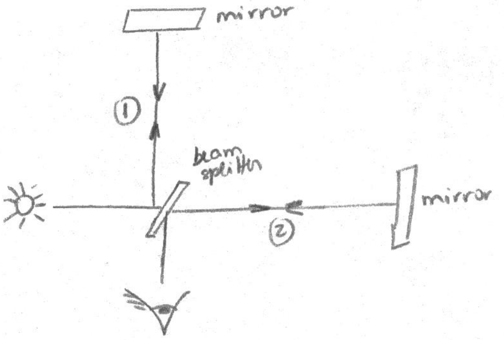
light to the right (+x):
$u'_x = u_x - v \Rightarrow u_{x'} = c - v$

light to the left (−x):
$\underset{\ominus}{u_{x'}} = \underset{-c}{u_x} - v \Rightarrow |u_{x'}| = c + v$

↑

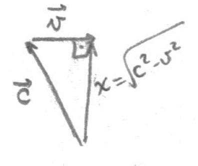
too small to measure directly
so they designed an
experiment based on
(interferometry)



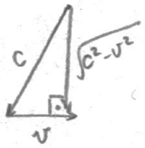


ⓒ is the speed of each boat in still water
c > v. Which boat (① or ②) wins the race?

$$t_{①} = t_{A\to B} + t_{B\to A} = \frac{L}{\sqrt{c^2-v^2}} + \frac{L}{\sqrt{c^2-v^2}} = \frac{2L}{c\sqrt{1-v^2/c^2}} = \frac{2L}{c}\underbrace{\left(1-\frac{v^2}{c^2}\right)^{-1/2}}_{\text{Taylor expansion}} = \boxed{\frac{2L}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}+\dots\right)}$$



A → B



$$(1-x)^{-1/2} \approx 1 + \left(-\frac{1}{2}(1-x)^{-3/2}(-1)\right)\Big|_{x\to 0} x + \dots$$

$$= 1 + x + \dots$$

$$x^2+v^2=c^2$$

$$x=\sqrt{c^2-v^2}$$

$$t_{②} = t_{A\to C} + t_{C\to A} = \frac{L}{c+v} + \frac{L}{c-v} = \frac{L(c-\cancel{v}+c+\cancel{v})}{c^2-v^2} = \frac{2L}{c^2\underbrace{\left(1-\frac{v^2}{c^2}\right)}} \simeq \boxed{\frac{2L}{c}\left(1+\frac{v^2}{c^2}+\dots\right)}$$

$$\left(1-\underset{x}{\left(\frac{v^2}{c^2}\right)}\right)^{-1} \underset{\text{Taylor}}{\simeq} 1 + \left(-1\left(1-\frac{v^2}{c^2}\right)^{-2}(-1)\right)\Big|_{x\to 0}\frac{v^2}{c^2} + \dots$$

$$t_{①} \simeq \frac{2L}{c} + \frac{Lv^2}{c^3} \quad , \quad t_{②} \simeq \frac{2L}{c} + 2L\frac{v^2}{c^3}$$

→ boat ① would win the race

→ Michelson's interferometer is based on the same idea, where the time difference translates into phase difference and consequent interference pattern

Shift expected from Michelson-Morley experiment was never detected

(!) There is no ether, there is no special frame in which light speed is ⓒ
electromagnetic waves propagate in the vacuum

Einstein's postulate

we need to modify Galilean transformation

① The laws of physics are the same in all inertial reference frames
↳ all inertial frames are equivalent for all phenomena (mech. or electromag)

② The speed of light in the vacuum is ⓒ for any inertial ref. frame
↳ all observers measure the same value ⓒ for the speed of light, independent of the motion of the source

Galilean transf ⇒ $t = t'$ ⇒ there is the same time scale at all places in any frame – universal time scale

Events and observers

•) An event occurs at some point in space and at some instant in time
→ is recorded in space and time

•) Events are described by observers who belong to particular inertial frames

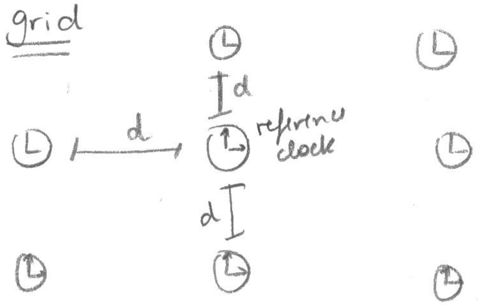
Simultaneity (defining time scale in a single frame)

•) simultaneous events: the arrival of the train and hand of watch points at 7
nearby

•) how to determine simultaneity of events at separated locations?
↳ need to synchronize clocks
↳ which involves transmission of signals
↳ with finite speed $v \leq c$

→) We CAN synchronize clocks in one reference frame

In one frame



·) reference clock sends a flash out light to other

·) d is known and also ⓒ

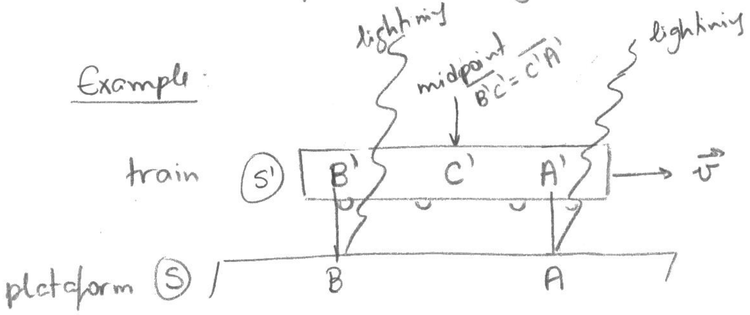
An event occurring at t_1 and x_1 is simulteneous with an event at t_2 and x_2 if light emitted from each arrives simultaneously at the midpoint between x_1 and x_2

↳ this definition mixes time and space

Relativity of simultaneity

Two spatially separated events simultaneous in one reference frame are not, in general, simultaneous in another inertial frame moving relative to the first.

Example:



·) lightining bolts strike A and B simultaneously

·) they reach C simultaneously

} events are simultaneous in Ⓢ

·) train is moving : signal from A' will reach C' before the signal from B'

↳ events are not simultaneous in (S')

→) we CANNOT synchronize clocks at different frames / time is not absolute

Lorentz transformation

↳ Einstein's bold move

Galilean transf. had not been questioned

↳ this would imply in changes in Newton's equations!

We want to make a change → this modifies the classical relation $\boxed{t=t'}$

$$x = \textcircled{\gamma}\,(\underline{\underline{x'}} + \underline{\underline{vt'}}) \quad , \quad x' = \textcircled{\gamma}\,(x - vt) \overset{(*)}{\Rightarrow}$$

$$x = \gamma(\gamma x - \gamma v t + v t')$$

$$\gamma v t' = (1-\gamma^2)x + \gamma^2 v t$$

$$(**) \quad \boxed{t' = \gamma t + \frac{(1-\gamma^2)}{\gamma}\frac{x}{v}} \quad (!)$$

so that Maxwell's equations are <u>invariant</u> and light propagates at speed ⓒ in any inertial frame

→) At $t=t'=0$ a flash of light starts at the origin of S and S'

S (spherical wave) S'

$$x^2+y^2+z^2 = c^2t^2 \qquad\qquad x'^2+y'^2+z'^2 = c^2t'^2$$

$$\swarrow (*)\,(**)$$

$$\gamma^2(x-vt)^2 + y^2 + z^2 = c^2\,\gamma^2\left(t + \frac{1-\gamma^2}{\gamma^2}\frac{x}{v}\right)^2$$

$$\underbrace{\left(\gamma^2 - c^2\frac{(1-\gamma^2)^2}{\gamma^2 v^2}\right)}_{1} x^2 + \underbrace{\left(-2\gamma^2 v t - c^2 2\frac{(1-\gamma^2)}{v}t\right)}_{=0} x + y^2 + z^2 = c^2t^2\underbrace{\left(\gamma^2 - \gamma^2\frac{v^2}{c^2}\right)}_{1}$$

$$\gamma^2\left(1-\frac{v^2}{c^2}\right) = 1 \Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} \,/\!/ \qquad \boxed{\beta = \frac{v}{c}}$$

$$\cdot)\quad t' = \gamma t + \frac{(1-\gamma^2)}{\gamma}\,\frac{x}{v} = \gamma\left(t + \underbrace{\frac{(1-\gamma^2)}{\gamma^2}}\,\frac{x}{v}\right)$$

$$\left(1 - \frac{1}{1-v^2/c^2}\right)\left(1-v^2/c^2\right) = \frac{\cancel{1} - v^2/c^2 - \cancel{1}}{} = -\frac{v^2}{c^2}$$

$$\Rightarrow t' = \gamma\left(t - \frac{v x}{c^2}\right)$$

to find t : $x = \gamma(x' + vt')$ into $x' = \gamma(x - vt)$

$$x' = \gamma^2 x' + \gamma^2 v t' - \gamma v t$$

$$t = \frac{(\gamma^2-1)\,x'}{\gamma v} + \frac{\gamma^{\cancel{2}}\cancel{v}\,t'}{\cancel{\gamma}\cancel{v}} \Rightarrow t = \gamma t' + \boxed{\frac{\gamma^2-1}{\gamma v}}\,x'$$

$$\frac{\cancel{1} - \cancel{1} + v^2/c^2}{\left(1-v^2/c^2\right)\;v}\sqrt{1-v^2/c^2} = \frac{v}{c^2}\,\frac{1}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow t = \gamma\left(t' + \frac{v x'}{c^2}\right)$$

$$\underline{\quad S \quad}$$

$$\begin{cases} x = \gamma(x' + vt') \\ y = y' \\ z = z' \\ t = \gamma\left(t' + \dfrac{v}{c^2}x'\right) \end{cases}$$

$$\underline{\quad S' \quad}$$

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \dfrac{v}{c^2}x\right) \end{cases}$$

Example 1-3 Two cosmic-ray muons are recorded by detectors in the lab (ref. frame Ⓢ), one at t_a at x_a and the other at t_b at x_b.

What is the time interval between those two events in (S') which moves at speed v relative to S ?

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\underbrace{t_b' - t_a'} = \gamma\left(t_b - t_a\right) - \frac{\gamma v}{c^2}\left(x_b - x_a\right)$$

depends on time interval in Ⓢ AND spatial separation of clocks in Ⓢ

→) PROPER TIME interval $\Big\}$ $\underbrace{(t_b - t_a)}_{\tau}$ when $x_b = x_a$

PROPER TIME : time interval measured in a frame where the events occur in the same place

For S' : •) PROPER TIME interval is the minimum time interval between those events

it occurs when $v = 0 \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1 \Rightarrow (t_b' - t_a') = (t_b - t_a)$

•) in general: $\gamma \geq 1$ and $\gamma \to \infty$ when $v \to c$

so $(t_b' - t_a') = \gamma(t_b - t_a) \geq (t_b - t_a)$

time interval is LONGER by a factor (γ) in a frame moving relative to the frame of the proper time

TIME DILATION

→) For which reference frame S'' would those events be simultaneous?

$$t_b'' - t_a'' = 0 \Rightarrow \gamma(t_b - t_a) = \frac{\gamma v}{c^2}(x_b - x_a)$$

$$\boxed{\frac{v}{c} = \frac{(t_b - t_a)\,c}{(x_b - x_a)}}$$

//

TIME DILATION

S' moves at v with respect to S

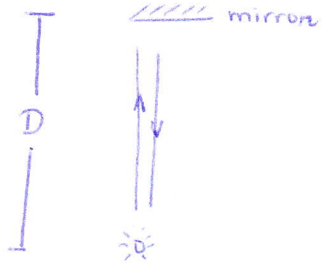
At t=0, they coincide

↓

Observer O' at rest in S' triggers a flash gun

In S'

time interval between emission and reception

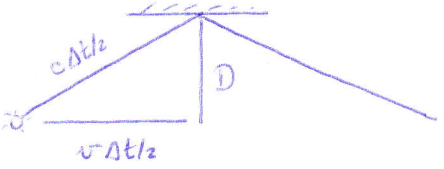


$$\Delta t' = \frac{2D}{c} \qquad \Delta t' = \tau$$

(proper time)

→) light travels at ⓒ in S and S'

In S



$$c^2\left(\frac{\Delta t}{2}\right)^2 = D^2 + v^2\left(\frac{\Delta t}{2}\right)^2$$

$$\Delta t^2 = \frac{4D^2}{c^2 - v^2} = \frac{4D^2}{c^2(1 - v^2/c^2)}$$

$$\boxed{\Delta t = \gamma \Delta t'}$$

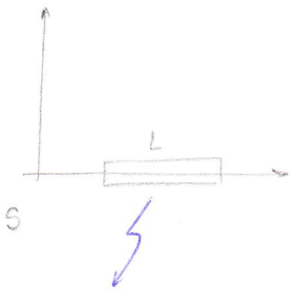
$\gamma \geq 1 \Rightarrow$ Time dilation

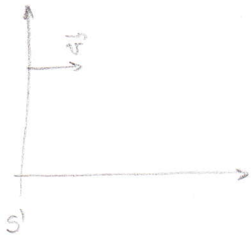
∘) light travels farther in S

∘) it takes longer in S to reach the mirror and return

13-(03)

Length Contraction





HW

Tipler
- 1-13a
- 1-19
- 1.23a
- 1-24
- 1.28

PROPER length

length of an object measured in the reference frame in which the object is at rest

What is L' as seen from an observer in S'?

$$L' = x'_b - x'_a = \gamma\left((x_b - x_a) - v\,\underbrace{(t_b - t_a)}\right)$$

no information about Δt

in S, the object is at rest, so we can measure its extremes x_b and x_a at any time

$$L = x_b - x_a = \gamma\left(\underbrace{(x'_b - x'_a)}_{L'} + v\,\underbrace{(t'_b - t'_a)}\right)$$

we have to measure x'_b and x'_a at the same time, because the object is moving, so $t'_b - t'_a = 0$

$$\boxed{L' = \frac{1}{\gamma} L}$$

length of the object is smaller when measured in a frame with respect to which it is moving

13-04

Muon Decay

Muons decay after 2.2 μs in a reference frame where they are at rest

If muons are detected at 9.000 m above the sea level, are they expected to be observed at sea level?
They move at 0.998 c toward the surface of Earth

$$\left\{ v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = 0.998c \times 2.2 \times 10^{-6} = \underline{660\text{ m}} \right.$$

so it might seem as the muons should decay before reaching sea level

BUT observations show that nearly all muons detected at 9.000m reach sea level

Events $\left\{ \begin{array}{l} \text{muon formed } t'_A \\ \text{muon decayed } t'_B \end{array} \right\}$ in $\boxed{\text{muon}}$ ref. frame $t'_B - t'_A = \tau$ proper lifetime

! CAREFUL muon lifetime is 2.2 μs in its own reference frame

For the Earth reference frame,
the muon lifetime is increased according to time dilation

⟹ Earth ref. frame

$\Delta t = \gamma \, \Delta t'$ $\qquad \gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} \simeq 15.8$

$\Delta t \simeq 35 \, \mu s$

↳ which explains why muons DON'T decay

atmosphere is at rest with respect to $\boxed{\text{Earth}}$. 9.000m is its proper thickness L

⟹ Muon ref. frame
↳ muons see atmosphere moving

L' contracts

$$L' = L/\gamma = \frac{9.000}{15.8} \approx \boxed{570\text{ m}}$$

↳ so the muons have no reason to decay
they have not travelled 660m yet

Example 1-8

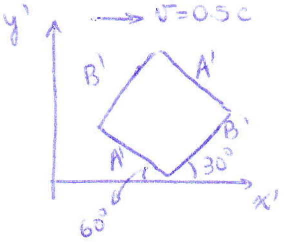
Elephants have a gestation period of 21 months.
A freshly impregnated elephants is sent toward space at $v = 0.75c$
If we monitor radio transmissions from the spaceship,
how long after launch should we find out about the
newborn calf?

Example 1-9

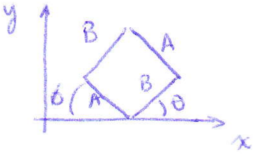
A stick that has a proper length of 1m moves in a
direction parallel to its length with speed v relative to you
the length of the stick as measured by you is 0.914m.
what is the speed v?

Example 1-10

Consider the square in the $x'y'$ plane of S' with one side
making a $30°$ angle with the x' axis. If S' moves with $\beta = 0.5$
relative to S, what is the shape and orientation
of the figure in S?



$A = ?$ $B = ?$ $\theta = ?$ $\phi = ?$



S' moves in the x direction of S $\Rightarrow$ contraction may happen in x
there is no motion in the y direction not in y

Example 1-8

S': reference frame of elephant

$\tau = 21$ months (proper gestation period)

S: reference frame of Earth

time dilation

↳ $\Delta t = \gamma \tau$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.75^2}} \simeq 1.51$$

① $\Delta t = 21 \times 1.51 \simeq \boxed{31.7 \text{ months}}$

② radio signal is sent to Earth

↳ speed ⓒ

it has to travel a distance $\Delta x = \gamma(\cancel{\Delta x'} + v\,\Delta t')$

elephant doesn't move in its ref frame

↳ proper time

$$\Delta x = \gamma v \tau$$

$$\Delta x = \gamma (\beta c) \tau = 1.51\,(0.75)\,(21)\,c$$

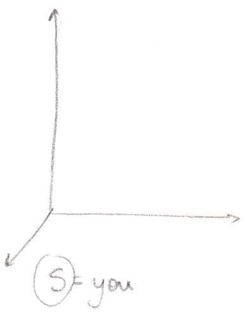
$$\boxed{\Delta x = 23.8\,c}$$

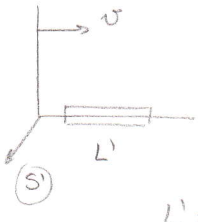
③ time for radio signal to reach Earth

$$\Delta t_s = \frac{\Delta x}{c} = \frac{23.8\,c}{c} = \boxed{23.8 \text{ months}}$$

④ total time for the news: $\Delta t + \Delta t_s = 31.7 + 23.8 = \boxed{55.5 \text{ months}}$ //

Example 1-9





(stick at rest in S')

$L' = 1\text{m}$
(proper length)

$L = 0.914\text{m}$

$$L = \frac{L'}{\gamma} = \underline{1\text{m}}\sqrt{1-v^2/c^2} \Rightarrow 1 - v^2/c^2 = (0.914)^2$$

$$\boxed{v = 0.406c}$$

Example 1-10

(square at rest in S') ⇒ contraction may happen for S

$$\left\{\begin{array}{lll} A_x = \dfrac{A_x'}{\gamma} & \Rightarrow & A_x = \dfrac{A'\cos 60°}{\gamma} \\ A_y = A_y' & \Rightarrow & A_y = A'\sin 60° \end{array}\right\} A = \sqrt{A_x^2 + A_y^2} = A'\sqrt{\frac{\cos^2 60°}{\gamma^2} + \sin^2 60°}$$

$$\boxed{A = 0.968A'}$$

$$\left\{\begin{array}{lll} B_x = B_x'/\gamma & \Rightarrow & B_x = B'\cos 30°/\gamma \\ B_y = B_y' & \Rightarrow & B_y = B'\sin 30° \end{array}\right\} B = \sqrt{B_x^2 + B_y^2} = B'\sqrt{\cos^2 30°/\gamma^2 + \sin^2 30°}$$

$$\boxed{B = 0.901B'}$$

$$\tan\theta = B_y/B_x = \frac{B'\sin 30°}{\cancel{B'}\cos 30°/\gamma} \Rightarrow \boxed{\theta = 33.7°}$$

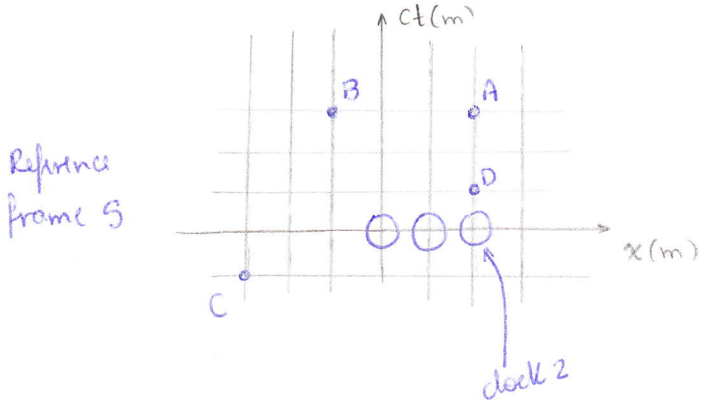
$$\tan\phi = A_y/A_x = \frac{\cancel{A'}\sin 60°}{\cancel{A'}\cos 60°/\gamma} \Rightarrow \boxed{\phi = 63.4°}$$

Spacetime Diagram

Events are fundamental

→ t vs x graphs

Let's ignore $y'=y$ and $z'=z$



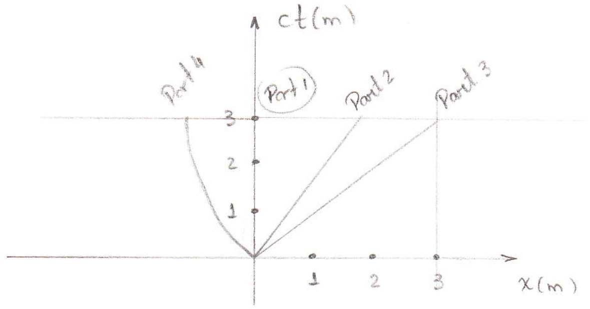
- events A and D occur at the same place at ≠ times

  $t_A - t_D$ → proper time, as measured by clock 2

- events A and B are simultaneous

- event C happened before the present (we consider $ct = ct' = 0$ to be the present

worldline: trajectory a particle on the spacetime diagram



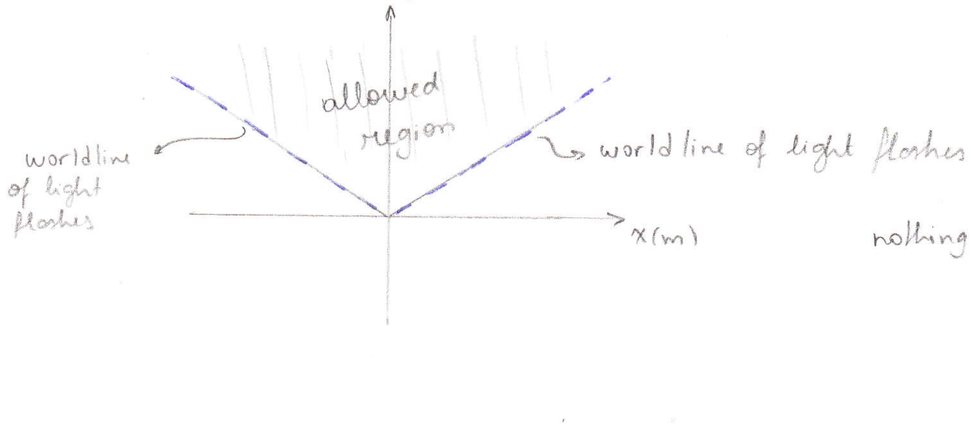
→) Particle 1 is at rest

→) Particle 2 moves at $u < c$

$u = \Delta x / \Delta t$

→) Particle 3 moves at ⓒ (light pulse)

→) Particle 4 slows down



nothing moves faster than light

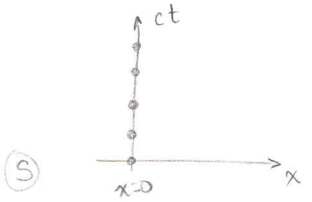
S' moves in the +x direction of S at v

They coincide at $t=t'=0$ $(x=x'=0)$

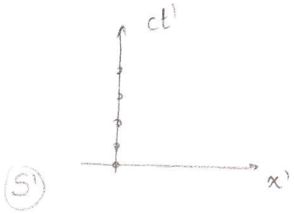
How does S' appear in the spacetime diagram of S ?

To find ct' axis in the spacetime diagram of S

→) Worldline of $x=0$ in S is ct axis



Worldline of $x'=0$ in S' is ct' axis



$\beta = \frac{v}{c} < 1$

→) The slope of ct' may be found from Lorentz transformation

$$x' = \gamma(x - vt)$$

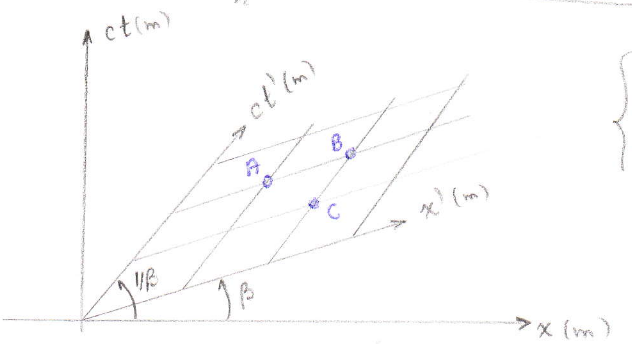
$$v = \frac{x}{t} = \frac{xc}{ct} \Rightarrow ct = \frac{x}{v/c} \Rightarrow \boxed{ct = \frac{1}{\beta}x}$$

→) To find x' axis in the spacetime diagram of S

x' axis is the axis for all points where $ct'=0$

use Lorentz transformation

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \Rightarrow t = \frac{v}{c^2}x \Rightarrow \boxed{ct = \beta x}$$



Events:

- A and B: simultaneous in S'
- B and C: same place in S'

Example 1-7

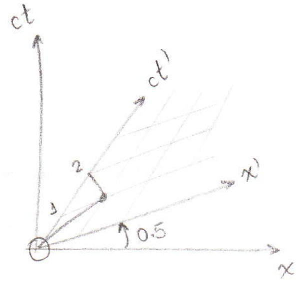
Two events occure at the same point x'_0 at times t'_1 and t'_2 in S', which moves with speed v relative to S. What is the spatial seperation of these events measured in S ?

$$\Delta x = \gamma\,(\Delta \cancel{x'} + v\,\Delta t') = \gamma\, v\, \tau$$

→) Using the figure { $\beta = 0.5$ (slope of x' axis)

↳ $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1.15$

$\tau = \dfrac{2}{c}$



$$\Delta x = \gamma\,\frac{v}{c}\,c\tau = \gamma\,\beta\,c\tau = (1.15)\,(0.5)\,(2)$$

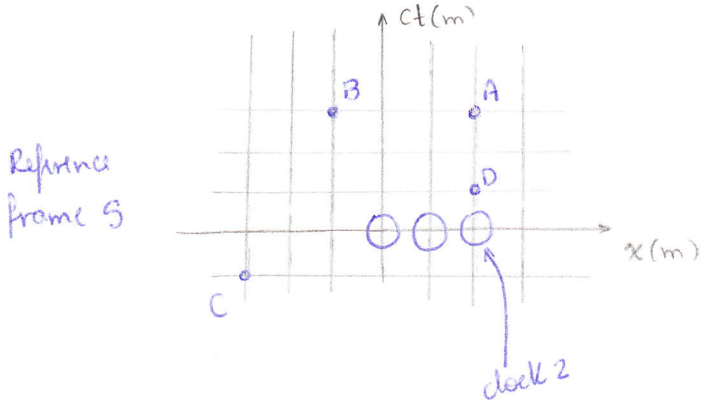
$\Delta x = 1.15\ \mathrm{m}$

Spacetime Diagram

Events are fundamental

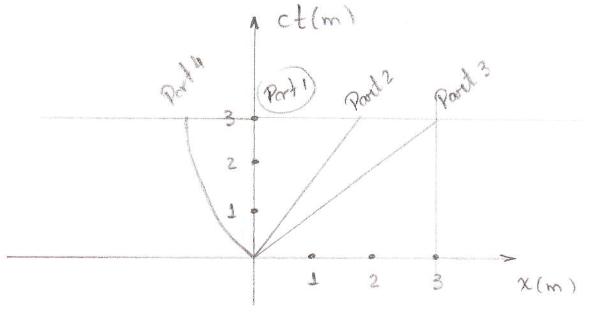
→ t vs x graphs

Let's ignore $y'=y$ and $z'=z$



- events A and D occur at the same place at ≠ times
  $t_A - t_D$ → proper time, as measured by clock 2
- events A and B are <u>simultaneous</u>
- event C happened before the present (we consider $ct = ct' = 0$ to be the present

<u>worldline</u>: trajectory a particle on the spacetime diagram

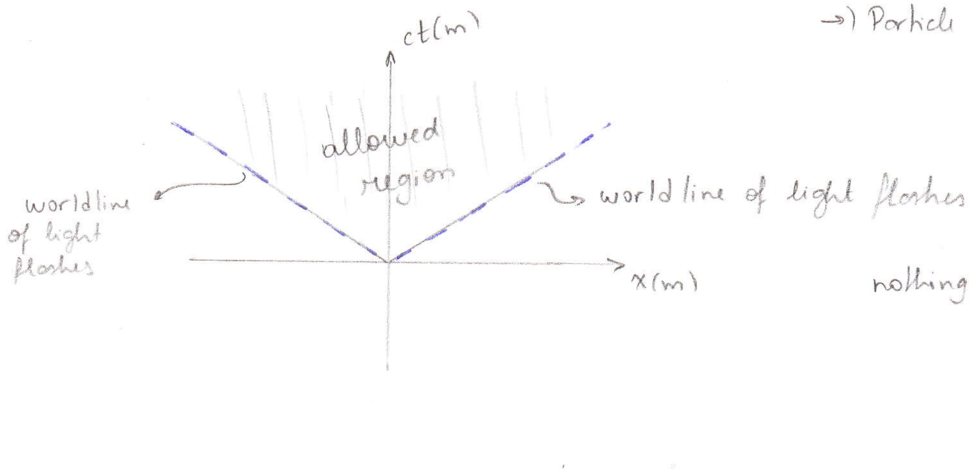


→) Particle 1 is at rest

→) Particle 2 moves at $u < c$
$u = \Delta x / \Delta t$

→) Particle 3 moves at ⓒ (light pulse)

→) Particle 4 slows down



nothing moves faster than light

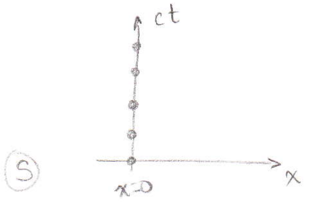
S' moves in the +x direction of S at v

They coincide at $t = t' = 0$ $(x = x' = 0)$

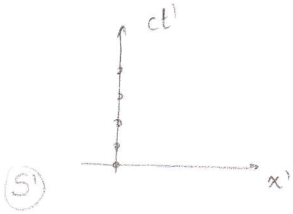
How does S' appear in the spacetime diagram of S ?

To find ct' axis in the spacetime diagram of S

→ Worldline of $x=0$ in S is ct axis



Worldline of $x'=0$ in S' is ct' axis



$\beta = \frac{v}{c} < 1$

→ The slope of ct' may be found from Lorentz transformation

$$x' = \gamma(x - vt)$$

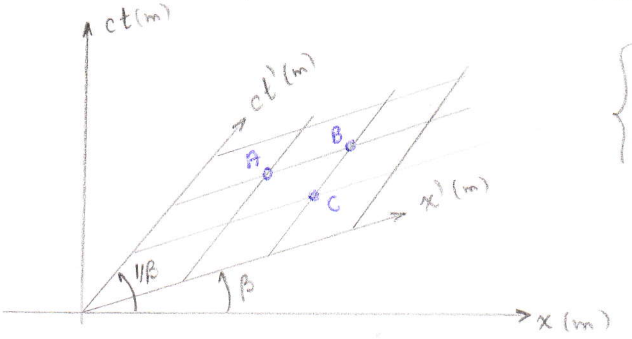
$$v = \frac{x}{t} = \frac{xc}{ct} \Rightarrow ct = \frac{x}{v/c} \Rightarrow \boxed{ct = \frac{1}{\beta} x}$$

→ To find x' axis in the spacetime diagram of S

x' axis is the axis for all points where $ct' = 0$

use Lorentz transformation

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \Rightarrow t = \frac{v}{c^2}x \Rightarrow \boxed{ct = \beta x}$$



Events:

- A and B: simultaneous in S'
- B and C: same place in S'

Example 1-7

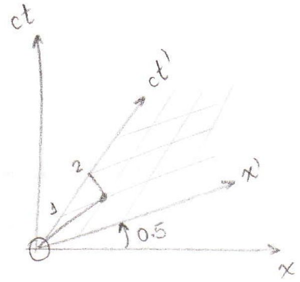
Two events occur at the same ~~[illegible]~~ point x'_0 at times t'_1 and t'_2 in S', which moves with speed v relative to S. What is the spatial seperation of these events measured in S?

$$\Delta x = \gamma\,(\Delta \cancel{x'} + v\,\Delta t') = \gamma v \tau$$

→) Using the figure: $\boxed{\beta = 0.5}$ (slope of x' axis)

↳ $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1.15$

$\boxed{\tau = \dfrac{2}{c}}$



$$\Delta x = \gamma \frac{v}{c} c\tau = \gamma \beta c \tau = (1.15)(0.5)(2)$$

$$\boxed{\Delta x = 1.15\ \text{m}}$$

Relativistic Mass

Lorentz transf. ⇒ need modification of the equations of mechanics so that they remain **INVARIANT** under the transformation from one inertial frame to another

⇒ **RELATIVISTIC mechanics**

Newton's 2nd law in the form $\vec{F} = m\vec{a}$ is (not) relativistically invariant

but

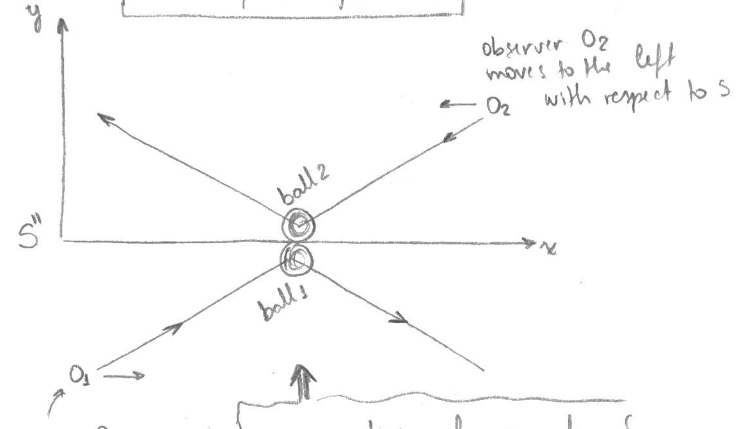
$\vec{F} = \frac{d\vec{p}}{dt}$ is relativistically invariant if the relativistic momentum $\vec{p}$ is used

$$\boxed{\vec{p} = \underbrace{m(v)}\vec{v}}$$

↳ mass is a function of v

Thought experiment

From reference frame S''

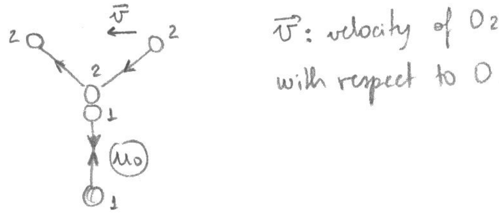


⇑

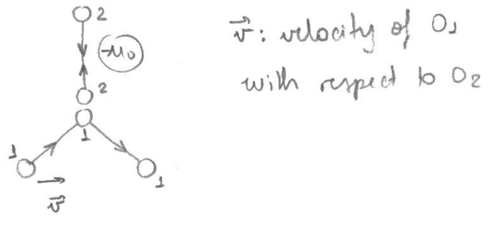
conservation of momentum

elastic collision

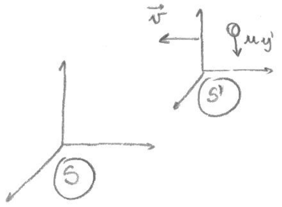
From the reference frame of O₁ } Ⓢ



From the reference frame of O₂ (S')



$\rightarrow$) From the reference frame of O_1 (S)



Ball 1 moves along the y axis with velocity $u_{y_1} = u_0$

Ball 2 has an x and y components of its velocity

Ball 2

$$u_{x_2} = \frac{u'_{x_2} + v}{1 + \frac{v}{c^2} u'_{x_2}} \xrightarrow{u_x' = 0} u_{x_2} = v$$

$$u_{y_2} = \frac{u'_{y_2}}{\gamma \left(1 + \frac{v}{c^2} u'_{x_2}\right)} \xrightarrow{u_y' = -u_0} u_{y_2} = \ominus \frac{u_0}{\gamma}$$

u_{y_2} is smaller than u_{y_1}, because time taken for ② to travel in S (O_1) is greater than measured in S' (O_2)

$$\underbrace{\text{Before collision} \quad u_{y_1} = u_0 \quad u_{y_2} = -\frac{u_0}{\gamma}}_{} \qquad \text{After collision} \quad u_{y_1} = -u_0 \quad u_{y_2} = \frac{u_0}{\gamma}$$

to guarantee conservation of momentum in S $\Rightarrow$ m(v)

(before collision) (after collision)

$$m(u_0)\, u_0 - m(u)\, u_{y_2} = -m(u_0)\, u_0 + m(u)\, u_{y_2}$$

$$\boxed{\frac{m(u)}{m(u_0)} = \frac{u_0}{u_{y_2}} = \frac{u_0}{u_0 \sqrt{1 - v^2/c^2}}}$$

Approximation : $u_0 << v \Rightarrow u_{y_2} = u_0 \sqrt{1 - \frac{v^2}{c^2}} << v$

since $u_{x_2} = v$

$$\boxed{u_2 \simeq v}$$

$u_0 \simeq 0$

$u_2 \simeq v$

$$\boxed{m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}}$$

m_0 is the classically measured mass of the particle (mass of the particle at rest)

mass of an object in motion with respect to an observer is LARGER than measured when it is at rest

→) Conservation of momentum is valid in relativity provided we write

$$\boxed{\vec{p} = \frac{m_0 \vec{u}}{\sqrt{1 - u^2/c^2}}}$$

mass

$$m(u) = \frac{m_0}{\sqrt{1 - u^2/c^2}}$$

→) in the reference frame of O_1 (S) $\begin{cases} p_{y_1} + p_{y_2} \text{ (before collision)} \\ \text{and} \\ p_{y_1} + p_{y_2} \text{ (after collision)} \end{cases}$ only changes signs, so for momentum to be conserved

$p_{y_1} + p_{y_2} = 0$

Proof

$$p_{y_1} = \frac{m_0 u_0}{\sqrt{1 - u_0^2/c^2}} \qquad p_{y_2} = \frac{m_0 u_{y_2}}{\sqrt{1 - (u_{x_2}^2 + u_{y_2}^2)/c^2}}$$

and so $p_{y_1} + p_{y_2} = 0$

showing it

$$p_{y_2} = -\frac{m_0 u_0 \sqrt{1 - v^2/c^2}}{\sqrt{1 - \left(v^2 + u_0^2 - u_0^2 \frac{v^2}{c^2}\right)/c^2}} = \frac{-m_0 u_0 \sqrt{1 - v^2/c^2}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right) - \frac{u_0^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}} = \frac{-m_0 u_0 \cancel{\sqrt{1 - v^2/c^2}}}{\cancel{\sqrt{1 - v^2/c^2}} \sqrt{1 - u_0^2/c^2}}$$

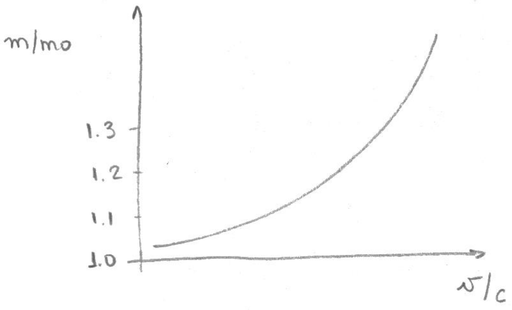
$$\boxed{p_{y_2} = -p_{y_1}}$$

Bucherer 1909

1st experimental confirmation of the dependence of mass on velocity

e^- of high velocity → technique similar to the one used by Thomson to measure q/m

↳ $quB = m\frac{u^2}{R} \Rightarrow \frac{q}{m} = \frac{u}{RB}$



prove that $m(v)$

and that $c = 2.998 \times 10^8$ m/s

Example 2-1

For what value of (u/c) will the measured mass of an object $\frac{m_0}{\sqrt{1-u^2/c^2}}$ exceed the rest mass by a fraction f?

↳ that is, $\frac{m_0}{\sqrt{1-u^2/c^2}} = m_0 + m_0 f$

$$f = \frac{1}{\sqrt{1-u^2/c^2}} - 1 \Rightarrow \frac{1}{(1-u^2/c^2)} = (f+1)^2 \Rightarrow 1-u^2/c^2 = \frac{1}{(f+1)^2}$$

$$u^2/c^2 = \frac{(f+1)^2-1}{(f+1)^2} \Rightarrow u/c = \frac{\sqrt{f^2+2f}}{(f+1)} \Rightarrow \boxed{u/c = \frac{\sqrt{f(f+2)}}{f+1}}$$ //

Example 2-2

A high-speed interplanetary probe with a mass $m = 50{,}000$ kg has been sent toward Pluto at a speed $u = 0.8c$. What is its momentum as measured by Mission Control on Earth? If, preparatory to landing on Pluto, the probe's speed is reduced to 0.4c, by how much does its momentum change?

$$p_{0.8c} = \frac{m_0 u}{\sqrt{1-u^2/c^2}} = \frac{50\times10^3 (0.8c)}{\sqrt{1-(0.8)^2}} = \boxed{2.0\times10^{13} \text{ kg m/s}}$$ //

$$p_{0.8c} - p_{0.4c} = 2.0\times10^{13} - \frac{50\times10^3 (0.4c)}{\sqrt{1-(0.4)^2}} = 2.0\times10^{13} - 6.5\times10^{12} = \boxed{1.6\times10^{13} \text{ kg m/s}}$$ //

Relativistic Energy

A particle is initially at rest in x_i with mass m_0

F moves it to x_f

work

$$W = \int_{x_i}^{x_f} F\,dx \qquad F = \frac{dp}{dt} = \frac{d(m(u)\,u)}{dt} = u\frac{dm(u)}{dt} + m(u)\frac{du}{dt}$$

$$W = \int_{x_i}^{x_f} \left(u\,\frac{dm(u)}{dt} + m(u)\,\frac{du}{dt} \right) dx$$

$$m(u) = \frac{m_0}{\sqrt{1 - u^2/c^2}} \Rightarrow m^2(u)\left(1 - u^2/c^2\right) = m_0^2$$

Let us call

the relativistic mass $\overline{\underline{m(u) = m}}$

$$m^2(1 - u^2/c^2) = m_0^2$$

$$m^2c^2 - m^2u^2 = m_0^2c^2$$

d/dt

$$\hookrightarrow c^2\frac{dm^2}{dt} - \frac{d(m^2u^2)}{dt} = 0$$

$$\cancel{2}c^2\cancel{m}\frac{dm}{dt} - \cancel{2}\cancel{m}u^2\frac{dm}{dt} - \cancel{2}um^{\cancel{2}}\frac{du}{dt} = 0$$

$$u^2\frac{dm}{dt} + um\frac{du}{dt} = c^2\frac{dm}{dt} \Rightarrow u\frac{dm}{dt} + m\frac{du}{dt} = \frac{c^2}{u}\frac{dm}{dt} = c^2\frac{dm}{\cancel{dt}}\frac{\cancel{dt}}{dx} = c^2\frac{dm}{dx}$$

$$W = \int_{x_i}^{x_f} c^2\frac{dm}{dx}\,dx = c^2\int_{m_i}^{m_f} dm = c^2(m_f - m_i) = \boxed{\frac{m_0c^2}{\sqrt{1 - u^2/c^2}} - m_0c^2} \;//$$

Classical law of energy conservation : total work = change in kinetic energy

$$\boxed{K = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} - \underbrace{m_0 c^2}_{\text{rest mass}}} \leftarrow \text{relativistic kinetic energy}$$

∘) If $u << c \Rightarrow \left(1-u^2/c^2\right)^{-1/2} \simeq 1 + \left[-\frac{1}{2}\left(1-\frac{u^2}{c^2}\right)^{-3/2}(-1)\right]_{\frac{u^2}{c^2}\to 0} u^2/c^2 + \dots$

$$\simeq 1 + \frac{1}{2}\frac{u^2}{c^2}$$

$$\downarrow$$

$$K \simeq \cancel{m_0 c^2} + m_0 c^2 \frac{1}{2}\frac{u^2}{c^2} - \cancel{m_0 c^2} = \boxed{\frac{1}{2} m_0 u^2} \leftarrow \text{classical kinetic energy}$$

Relativistic total energy (definition)

$$E = K + m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} \qquad \boxed{E = mc^2} \qquad \left(m = \frac{m_0}{\sqrt{1-u^2/c^2}}\right)$$

work done by a net force increases the energy of the system from the rest energy $m_0 c^2$ to $\frac{m_0 c^2}{\sqrt{1-u^2/c^2}}$

∘) For $u << c \Rightarrow E = \frac{1}{2} m_0 u^2 + m_0 c^2$

(!) NOT in conflict with classical physics
Zero of energy is arbitrary so
we can add this constant $(m_0 c^2)$ to it.

$$E = mc^2 \qquad\qquad E = \frac{m_0}{\sqrt{1-u^2/c^2}}\,c^2$$

It is common to express the <u>masses</u> of particles in <u>energy units</u>

$$\underbrace{1.0\ \text{eV}}_{\text{amount of energy for}} = \underbrace{1.602\times10^{-19}\ \text{C}}_{\text{an electron to accelerate}} \times \underbrace{1.0\ \text{V}}_{\text{through a potential difference of 1 volt}} = 1.602\times10^{-19}\ \text{J}$$

Example: mass of electron at rest $= \boxed{9.11\times10^{-31}\ \text{kg}}$

Its rest energy is: $E = m_0c^2 = 9.11\times10^{-31}\ \text{kg}\cdot c^2 = 8.19\times10^{-14}\ \text{J}$

$$E = \frac{8.19\times10^{-14}\ \text{J}}{1.602\times10^{-19}\ \text{J}}\ 1\ \text{eV} = 5.11\times10^{5}\ \text{eV} \Rightarrow E = 0.511\ \text{MeV}$$

rest energy of the electron

$$\hookrightarrow\ m_0 = \frac{E}{c^2} = \boxed{0.511\ \text{MeV}/c^2}$$

mass of the electron at rest

Example 2.5

Muon speed relative to Earth = 0.998c. If its rest energy is 105.7 MeV, what will observers on Earth measure for its total energy? What will they measure for its mass?

$$E_{\text{rest}} = m_0c^2 = 105.7\ \text{MeV} \Rightarrow E = \frac{m_0c^2}{\sqrt{1-u^2/c^2}} = \frac{105.7}{\sqrt{1-(0.998)^2c^2/c^2}} = \boxed{1670\ \text{MeV}}$$

$$m = \frac{E}{c^2} = \boxed{1670\ \text{MeV}/c^2}$$

It is often convenient to have an expression for the total relativistic energy in terms of momentum

$$E^2 - m_o^2 c^4 =$$

$$m^2 c^4 - m_o^2 c^4 = m_o^2 c^4 \left(\frac{1}{1-u^2/c^2} - 1 \right) = \frac{m_o^2 c^4 \; u^2/c^2}{1-u^2/c^2}$$

$$= \frac{m_o^2 c^2 u^2}{1-u^2/c^2} = c^2 p^2$$

$$\Rightarrow m^2 c^4 = c^2 p^2 + m_o^2 c^4$$

$$\boxed{E^2 = c^2 p^2 + m_o^2 c^4}$$

Example 2-9 A particular object is observed to move through the lab at high speed. Its total energy $E = 4.5 \times 10^{17}$ J and $p_x = 3.8 \times 10^8$ Kg m/s, $p_y = 3.0 \times 10^8$ Kg m/s, $p_z = 3.0 \times 10^8$ Kg m/s. What is the object's rest mass?

$$m_o^2 = \frac{E^2}{c^4} - \frac{p^2}{c^2} = \left(\frac{4.5\times10^{17}}{c^2}\right)^2 - \left[\left(\frac{3.8\times10^8}{c}\right)^2 + \left(\frac{3.0\times10^8}{c}\right)^2 + \left(\frac{3.0\times10^8}{c}\right)^2\right]$$

$$\boxed{m_o = 4.6 \text{ Kg}}$$

Example 2-10 The total energy of an e^- produced in a reaction is 2.40 MeV. Find the e's momentum and speed. ($m_o = 9.11\times10^{-31}$ Kg, rest energy = 0.511 MeV)

- $pc = \sqrt{E^2 - m_o^2 c^4} = \sqrt{(2.40)^2 - (0.511)^2} \Rightarrow \boxed{p = 2.34 \text{ MeV/c}}$

- $E = \dfrac{m_o c^2}{\sqrt{1-u^2/c^2}}$ and $p = \dfrac{m_o}{\sqrt{1-u^2/c^2}} u \Rightarrow E = \dfrac{p c^2}{u} \Rightarrow$

$$\Rightarrow \boxed{u = \frac{pc}{E} c} \Rightarrow u = \frac{2.34}{2.40} c \Rightarrow \boxed{u = 0.975c}$$

Massless Particles

$m_0c^2 = 0$ → idea of zero rest mass has no analog in classical physics

$$\Downarrow$$

$$\boxed{E = pc} \qquad \text{and from } \frac{u}{c} = \frac{pc}{E} \Rightarrow \boxed{u = c}$$

A particle whose mass is zero moves at the speed of light

(photon, gluon, graviton)

Creation and Annihilation of particles

Equivalence of mass and energy ⇒

- particles combine with antiparticles and masses of both convert to energy
- mass can be created from energy

17-06

Mass / Energy Conversion

→) mass of an object ≠ mass of its constituent parts

∘) for atom, difference between its mass and the mass of its nucleus and e^- is usually small

BUT

∘) difference between the mass of a nucleus and its constituent parts (p, n) is important

In relativity

Internal energy of a system is greater than 2m by $\underbrace{E_k/c^2}_{\text{additional energy stored by the object}}$

when particles collide and lock } spring compresses and store energy

Fission of a ^{235}U nucleus

the energy released as kinetic energy of the fission fragments is a fraction of the rest mass of the original nucleus

^{235}U is excited by the capture of a Ⓝ (neutron)

^{236}U is unstable and splits into two nuclei, releases energy

AND emits several neutrons
than can produce fission in other nuclei } chain reaction

$$^{235}U + n \longrightarrow {}^{92}Kr + {}^{142}Ba + 2n + 179.4\,MeV$$

Particle Physics

1897 – Thomson – electron (Nobel Prize in Physics 1906)

1905 – Einstein – photon (Nobel Prize in Physics 1921)

1911 – Rutherford – atomic nucleus – hydrogen → proton (Nobel in Chemistry 1908 ↓ radioactivity)
(Thomson's student) (lightest nucleus)

periodic table : atomic mass increases more-rapid than nuclear charge

1932 – Chadwick – neutron (Nobel in Physics 1935)
(Rutherford's student)

(!) **Marie Curie**
↓
theory of radioactivity → Nobel Prize in Physics 1903 (Becquerel, Pierre Curie, Marie Curie)

radium
polonium } Nobel Prize in Chemistry 1911 (Marie Curie)

o) Walter Bothe and Herbert Becker (1928)

$$\text{Po} \xrightarrow{\alpha} \text{Be} \xrightarrow[\text{neutral radiation}]{\text{electrically}} ?$$

bombarded beryllium with α particles emitted from polonium. Be then gave off a penetrating electrically neutral radiation.
↙ used in Rutherford's experiment (nucleus of He)

(!)
o) **Irene Joliot-Curie** and Frederic Joliot-Curie (1932) → Nobel Prize in Chemistry 1935 } for artificial radioactivity NOT this experiment

continued Bothe's experiment

this radiation emitted protons from paraffin (hydrogen atoms) { analogy with Compton effect, in which x-ray photons in metal eject e^-, they concluded this radiation was γ-rays ejecting protons

•) Chadwick repeated the experiment (Nobel Prize in Physics 1935)

bombarded not only H in paraffin

but also He, N and other elements

proved beryllium emissions had mass ~ mass of proton

neutron

Neutron – impt role in nuclear reaction – easier to penetrate nucleus – no Coulomb repulsion

fission $^{235}U + n \rightarrow {}^{92}Kr + {}^{142}Ba + 2n + 179.4 MeV$

↓ of uranium

1938 by Otto Hahn and Fritz Strassmann

↑ (Rutherford's student)

only he got the Nobel Prize 1944

↓ experiments started with Lise Meitner (!)

who gave the interpretations from Sweden

Side Note

(!) Maria Goeppert-Mayer

Nobel Prize in Physics 1963

shell model

worked for many years with no salary

For a brief time:

$\underbrace{e^-, p, n, \text{photons}}_{\text{elementary particles}}$

But soon with particle accelerators producing greater and greater energies

an enormous amount of particles were discovered

↓

which are the elementary particles?

(p) and (n) – known as nucleons are not

scientists tried to put together the pieces of the puzzle in what they called

Standard model

Side note

•) Rutherford was Thomson's student

•) Bohr, Chadwick, Geiger, Hahn, etc. were Rutherford's students

Elementary Particles → (Standard Model)

they are either **bosons** or **fermions** depending on their spin

- bosons ↑ integer spin (particles associated with fundamental forces)
- fermions ↑ half-integer spin {Pauli exclusion principle} (particles associated with matter)

⇛ **Fermions**: (Table 12-1)

leptons (spin 1/2)		(charge)	quarks (spin 1/2)		(charge)
electron	e	-1	up	u	$2/3$
electron neutrino	ν_e	0	down	d	$-1/3$
muon	μ	-1	charm	c	$2/3$
muon neutrino	ν_μ	0	strange	s	$-1/3$
tau	τ	-1	top	t	$2/3$
tau neutrino	ν_τ	0	bottom	b	$-1/3$

$\underbrace{u \; d \; c \; s \; t \; b}_{\text{flavors}}$

strength	range (m)				
1	∞	gravitational force	✓	✓	
10^{36}	∞	electromagnetic force	✓	✓	
10^{25}	10^{-18}	weak interaction	✓	✓	{associated with radioactivity} Ex: beta decay
10^{38}	10^{-15}	strong interaction	✗	✓	{attractive force that holds nucleons together}

→) Quarks and antiquarks have never been detected as free particles

↳ evidence for their existence:

high energy electrons are deflected by protons through large angles

→) Analogous to electric charge, quarks have COLOR CHARGE (3 possible values): red, blue, green

Ex: there are 3 different u quarks: u_r, u_b, u_g

Every particle has a corresponding **antiparticle** with the same mass but opposite electric charge } QED quantum electrodynamics

Feyman, Schwinger, Tomonaga
Nobel Prize 1965

photon is its own antiparticle
particle + antiparticle may annihilate
antineutron is made of antiquarks
antiproton + positron → antihydrogen atom

→ why is the universe almost entirely made of matter and not antimatter?
↳ ongoing research
CP violation helps understanding it (Nobel Prize 1980 James Cronin Val Fitch)
C: charge
P: parity

Therefore,

leptons → anti leptons
quarks → antiquarks

Hadrons → bound states of the quarks and antiquarks

Baryons (3 quarks)
proton: uud
neutron: udd
Λ, Σ,

Mesons (quark + antiquark)
$\boxed{\text{pion}}$ $\pi^+ (u\bar{d})$, $\pi^0 (d\bar{d}, u\bar{u})$, $\pi^- (d\bar{u})$
Kaon
eta

spin 0
Yukawa predicted their existence
Nobel in Physics (1949)
was thought to be carriers of strong interaction

Pions were thought to be strong force mediators

Strong force mediators: gluon

⇛ **Bosons** → mediators of fundamental forces

↓

process by which elementary particles interact with each other

interaction – described as a physical field mediated by exchange of virtual particles { detectable as forces not as real particles

↓

(gauge bosons)

Elementery Bosons

- Gauge bosons
  - gluon, W and Z bosons, photon
- Higgs boson → undiscovered – Large Hadron Collider (LHC) CERN 2009
  - ↓ (mass arises from interaction with Higgs boson)
- graviton → not discovered

Interaction	current theory	mediators	strength	range (m)
strong	quantum chromodyn QCD	gluons	10^{38}	10^{-15}
electromagnetic	quantum electrodyn QED	photons	10^{36}	∞
weak	electroweak theory	W and Z bosons	10^{25}	10^{-18}
gravitation	general relativity	gravitons (not discovered)	1	∞

→) Electromagnetic interaction

↳ due to electric charge

Every charge is continually emitting and absorbing VIRTUAL photons
(not directly observed)

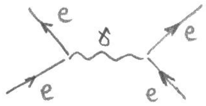
Charge can emit a virtual photon of energy $h\nu$
without violating cons. of energy or momentum
provided it exists for no longer than $\Delta t = \hbar / \Delta E$
Virtual photon can travel

$$R = c\Delta t = c\hbar / \Delta E = c\hbar / h\nu = \frac{\cancel{c}\,\cancel{h}\,\lambda}{2\pi\,\cancel{h}\,\cancel{c}} = \frac{\lambda}{2\pi}$$

and be absorbed by 2nd charge
which can emit virtual photon which is absorbed by 1st charge, ...

→ this exchange of virtual photons is the origin of the Coulomb force between the two charges

Feynman diagram



Coulomb repulsion between two e^-

→) Strong interaction → all hadrons interact via strong interaction

↳ due to color charge (leptons don't have color charge)

↳ mediators: gluons

→) Weak interaction → leptons and quarks participate

↳ mediators: W^+, W^-, Z^0 bosons (s=1)

interactions mediated by $W^\pm$ change quark flavor

Flavor – is a quantum number

↳ conserved number associated with conserved quantity

Ex: energy conservation → principal quantum number (n)

there are quantum numbers associated with { angular momentum, spin

↓

leptons: lepton number (L=1)

quarks: baryon number (B=1/3)

electric charge

} always conserved

↓ example

can know and explain which particles may or not be created from collisions and in decays

other quark flavors → conserved by strong interaction

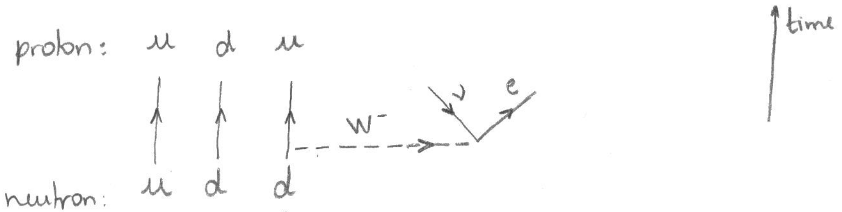
other quark flavors → violated by weak interaction

↓

strangeness,
charm,
bottom, etc

Free nucleon decay ↳ occurs via weak interaction because { strong force and electromagnetism cannot change flavor

$n \longrightarrow p + \bar{\nu}_e + e^-$



↳ emits a W^- and changes to u quark and W^- decays to e^- and $\bar{\nu}_e$

What conservation laws (if any) are violated by the following reactions?

a) $n \longrightarrow p\,\pi^-$

✓ no leptons – no prob. with cons. of lepton number

✓ net charge is zero before and after – charge is conserved

✓ baryon number is conserved, $B = +1$ before and after

✗ **BUT** energy is NOT conserved

rest energy of proton (938.3 MeV) + pion (139.6 MeV) > neutron (939.6 MeV)

b) $\Lambda^0 \longrightarrow \bar{p}\,\pi^+$

✓ no leptons

✓ charge is zero

✗ does NOT conserve baryon number

$B(\Lambda^0) = +1 \qquad B(\bar{p}) = -1 \qquad B(\pi^+) = 0$

O✓//

c) $\pi^- \rightarrow \mu^-\,\bar{\nu}_\mu$

✓ no baryons

✓ charge ⊖1

✓ rest energy of π^- (139.6 MeV) > μ^- (105.7 MeV) + $\bar{\nu}_\mu$

OK → difference appears as kinetic energy of the muon and neutrino

✓ lepton number

$L(\pi^-) = 0 \qquad L(\mu^-) = 1 \qquad L(\bar{\mu}_\nu) = -1$

↑ not lepton

◦) Main name behind conservation laws and symmetries

(!) $\boxed{\text{Emmy Noether}}$

↓

theorem 1918: every conservation law is a consequence of a symmetry

(!) $\boxed{\text{Chien - Shiung Wu}}$

↳ violation of parity

Chen Nin Yang and Tsung-Dao Lee } → Nobel in Physics 1957

Check also: {
Sophie Germain (1776 - 1831)
Émilie du Châtelet (1706 - 1749)
Hypatia (370? - 415)

www.agnesscott.edu/Lriddle/women/women.htm